

## Measurement Uncertainty and Significant Figures

There is no such thing as a *perfect* measurement. Even doing something as simple as measuring the length of an object with a ruler is subject to limitations that can affect how close your measurement is to its true value. For example, how clear and accurate is the scale on the ruler? How wide are the centimeter markings on the ruler? Can you trust that each centimeter marking represents *exactly* one centimeter? It's almost certainly not perfect, though it's probably pretty close. Companies that make inaccurate rulers probably won't stay in business very long.

The degree to which a measured quantity compares to the true value of the measurement describes the **accuracy** of the measurement. Most measuring instruments you will use in physics lab are quite accurate when used properly. However, even when an instrument is used properly, it is quite normal for different people to get slightly different values when measuring the same quantity. For example, when using a ruler, perception of when an object is lined up exactly against the ruler scale may vary from person to person. Sometimes a measurement must be taken under less than ideal conditions, such as at an awkward angle or against a rough surface. As a result, if the measurement is repeated by different people, or even by the same person, the measured value can vary slightly. The degree to which repeated measurements of the same quantity differ describes the **precision** of the measurement.

Because of limitations both in the accuracy and precision of measurements, you can never be certain that a measurement is exact. Because of this, in physics lab you are going to record measurements as a range of values within which you are pretty confident the exact value lies. Values will be recorded as

$$\text{best value} \pm \text{uncertainty}$$

where *best value* is your best estimate of the exact value and *uncertainty* is the maximum amount by which you think your measured value might differ from the exact value.

### I. Uncertainty and Accuracy

When performing a single measurement, your uncertainty should reflect the overall quality of the measurement you take. It is affected both by the resolution of the measuring instrument's scale and the conditions under which you are performing the measurement. Instruments such as pan balances are designed to measure mass to a certain decimal place with a given uncertainty. The pan balances in our lab are designed to be read to the nearest 0.01 grams  $\pm 0.01$  gram, so the uncertainty in a single measurement is 0.01 gram. Micrometer calipers, like pan balances, are designed to provide very accurate measurements. Under ideal conditions micrometer calipers can be used to measure thicknesses of objects to one micrometer ( $=0.001$  mm) with an uncertainty of 2 micrometers.

Other measuring tools, such as wooden metersticks or tape measures, are not designed to provide measurements as accurate as those from a micrometer caliper. In physics 107 labs, you will use tools like metersticks to provide good estimates of lengths of about half a meter or greater. Uncertainty using a meterstick, understandably, will be greater than when using a more accurate device, like a micrometer caliper. It is evident why this is so when you just look at a meterstick: the lines indicating the scale are thicker than on either a caliper or even a good ruler, making it hard to know exactly where to align the meterstick for measurement, and the wood of the meterstick itself may be slightly warped due to environmental influences. Because of these concerns, establishing uncertainty when using a meterstick is somewhat subjective, but the uncertainty should probably never be less than 1 mm. When deciding on uncertainty with this type of device, particularly if the measurement must be made under non-ideal conditions, some common sense thinking is the best way to proceed. The uncertainty you decide on needs to be a reasonable estimate of the amount your measurement might differ from the ideal value. With a meterstick, factors to consider include not only the clarity of the scale on the meterstick, but also

the ease of establishing the best start and endpoints for your measurement. If you are measuring the distance between two points marked on paper, are the points really one dimensional or do they have a measurable width? Do you measure center-to-center or from the inside edges of the points? Are the points blurry so that the edge of the point is difficult to establish? Such considerations play an essential part in determining uncertainty.

Because of these considerations, it is not unusual to have an uncertainty using a meterstick of 2 mm or more if the measurement is tricky to perform. This uncertainty may seem like a lot, but it is perfectly acceptable if the meterstick is being used appropriately. For example, you should not use a meterstick to measure lengths of just a few centimeters: even an inexpensive plastic ruler will provide a more accurate measurement on this scale. As an illustration, an uncertainty of 2 mm is 10% of a 2 cm measurement, which is excessive. On the other hand, a 2 mm uncertainty is less than 0.5% of a measurement of half a meter, which is acceptable.

**For our purposes in lab, all uncertainties should contain only one significant figure** (such as  $\pm 0.001$  m,  $\pm 10$  m/s or  $\pm 0.5$  s). It's fairly obvious why this should be the case: if you indicate an uncertainty of  $\pm 0.15$  m, for example, you are saying you are uncertain of your best value in the tenths place (the 0.1), but you are then indicating you know something about the hundredths place. If you are uncertain about the tenths place, how can you know anything about the hundredths place? Obtaining more than one significant figure in uncertainty happens most often when the uncertainty is established as the result of a calculation. If this happens, round the uncertainty to one significant figure: in this example the 0.15 m should round to 0.2 m (not 0.20 m, because you are indicating with the last zero that you know something about the hundredths place again!!)

When an uncertainty is established for a given measurement, as discussed above, the measurement is written as best value  $\pm$  uncertainty. As an example, I measure the width of a lab bench and obtain 0.67 m. The measurement is reasonably easy to do so I decide that the uncertainty is  $\pm 1$  mm (or 0.001 m). I need to be careful about how I write the value; if I were to write  $0.67 \text{ m} \pm 0.001 \text{ m}$  I am sending a mixed message about the measurement: the 0.67 m best value says I can measure the length to the nearest 0.01 m and not to any further decimal places. The uncertainty, however, indicates that the value should be accurate to somewhere in the 0.001 m decimal place. Which is correct? You can't really tell if you didn't perform the measurement. Therefore, when you write a measurement, you'll want to avoid this problem. **When you decide on the uncertainty, you are deciding how many decimal places the best value should contain: it should go to the same decimal place as the uncertainty.** Therefore, if the uncertainty in this measurement is 0.001 m, the best value should be read to 0.67# m, where # is your best estimate of the digit in the decimal place containing uncertainty.

Finally, a request. From time to time, people seem to have lists of different measurements made with the same device that are recorded similarly to the following:  $0.50 \text{ g} \pm 0.01 \text{ g}$ ,  $0.60 \text{ g} \pm 0.01 \text{ g}$ ,  $1.50 \text{ g} \pm 0.01 \text{ g}$ ,  $1.20 \text{ g} \pm 0.01 \text{ g}$ . What do these measurements have in common? The last digit of the best value in each is zero. While possible, this is statistically VERY unlikely. The first thing that comes to mind when we see this is that the measurers forgot to estimate the last digit, then filled in a bunch of zeros to make the measurements agree with the uncertainty. It's not that we're suspicious, but often it's kind of obvious that this happened, like when the zeros are added in pen when the best values were written in pencil. Please don't arouse our suspicions – be sure to make proper measurements!!

## II. Uncertainty and Precision

In physics 107 labs we will often establish uncertainty by performing several measurements of the same quantity. We suggest that each group member performs an independent measurement. It is important when performing independent measurements that group members **not** share measurement

information until all group members have performed the measurement (we don't want to hear "I got 2.7 cm – now see what you get" . . . "yeah, I get 2.7 cm too" . . . ). While it's human nature that group members want to agree on all measurements, it is only when group members obtain independent (often differing) measurements that the precision of the measurement can be determined. Further, determining the uncertainty of a measured value by comparing multiple trials often brings to light any problems you may not have noticed when performing a single measurement ("Why is your measurement 4 mm longer than mine . . . oh, did you measure to the edge or the center of the dot . . ."). This is often evident when measuring time using digital stopwatches. If the watch reads to 0.001 s, you might decide that an uncertainty of  $\pm 0.001$  s is valid for a single-measurement uncertainty. However, if three people perform the measurement simultaneously, their results might differ by a quarter of a second or more – hardly within your  $\pm 0.001$  s limit. This forces you to think about the many reasons for the difference: how well the stopwatches are calibrated, how much of a factor is human reaction time, and how clearly evident are the points at which the timing should start and stop are a few possible ideas.

When calculating uncertainty using measurements from multiple trials, we do so by averaging the measured values to obtain the best value, then using half of the difference between the maximum and minimum measured values to obtain the uncertainty. For example, in your lab group, you perform three independent trials of the same length measurement and record them as 2.7 cm  $\pm$  0.1 cm, 2.9 cm  $\pm$  0.1 cm and 3.0 cm  $\pm$  0.1 cm. The best value will be the average of the three individual values, or 2.867 cm (we'll worry about rounding later) and the uncertainty will be  $(3.0 \text{ cm} - 2.7 \text{ cm})/2 = 0.15 \text{ cm}$ . If you want to record the final result as 2.867 cm  $\pm$  0.15 cm you need to think again; above we indicated that the uncertainty must contain only one significant figure and the best value must be rounded to the same decimal place as the uncertainty. For our measurement, then, we must round the uncertainty from 0.15 cm to 0.2 cm and the best value must then be rounded to the tenths place. The final result, then, should be 2.9 cm  $\pm$  0.2 cm.

In most cases in physics 107 labs, the uncertainty obtained from multiple measurements will be larger than the uncertainty established from a single measurement; in this case the multiple measurement uncertainty will be used as the overall uncertainty. However, in that event that the uncertainty of the single measurement is larger (for example, in the rare event that group members independently obtain the same measured value), this single measurement uncertainty should be used.

### III. Significant Figures

Use of proper rules for significant figures (sigfigs) and mathematical operations with sigfigs can be time consuming AND are of limited benefit for us in what we are trying to learn in physics 107 labs. That being said, we need to recognize that while we're not going to be TOO picky about exact rules for sigfigs, we aren't going to abuse them either. For example, I walk a distance of 4.58 m in 7.0 seconds. I calculate my average speed by dividing distance by time:  $4.58 \text{ m}/7.0 \text{ s} = ??$ . My calculator tells me the result of this operation is 0.654285714286. If I write this value as my final result (in m/s) I am indicating that I am certain of my speed to 1 thousand billionth of a meter per second. This constitutes sigfig abuse. The distance 4.58 contains 3 sigfigs and 7.0 s contains two. While rules of sigfigs would specify the final result should contain two sig figs, we are only concerned that you recognize the result should be to a few sig figs – not one sigfig and not however many your calculator gives you. Expressing this value as 0.65 m/s or 0.654 m/s is fine; 0.7 m/s is pretty minimal and 0.654285714286 m/s is totally unacceptable. The final result you give should show you are aware of the fact that it was produced using measurements that have a reasonable limit to their accuracy.