Lesson Two: Math Blackboard, Logic, Quantifiers and Alignment

Math Blackboard Symbols

We use `\mathbb` to display some common sets of numbers.
naturals: $\mathbb{N}$
rationals: $\mathbb{Q}$
reals: $\mathbb{R}$
positive reals: $\mathbb{R}^+$

Logic and Quantifiers

The propositions $P \Rightarrow Q$ and $(\sim P) \vee Q$ are equivalent.
The proposition $R \wedge (\sim R)$ is a contradiction; it is always false.
The negation of $(\forall n \in \mathbb{N})(n \text{ is prime})$ is $(\exists n \in \mathbb{N})(n \text{ is not prime})$.

Alignment

We can also do negations step by step. To make things line up nicely, we use LaTeX's built-in "align" environment.

$$\begin{aligned}
\sim [(z \text{ is odd }) \vee (z \text{ is even})] &\iff [\sim (z \text{ is odd })] \wedge [\sim (z \text{ is even})] \\
&\iff (z \text{ is not odd}) \wedge (z \text{ is not even})
\end{aligned}$$

Exercise Two: Math Blackboard, Logic, Quantifiers and Alignment

The proposition $(P \vee Q) \vee (\sim P \wedge \sim Q)$ is a tautology because it is always true.

[`To typeset some of the following symbols, you may need to use the Help menu. Also, try using the \not command`]

$$\begin{aligned}
\sim[(\exists n \in \mathbb{N})(\forall m \in \mathbb{N})(n \leq m)] &\iff (\forall n \in \mathbb{N})\sim[(\forall m \in \mathbb{N})(n \leq m)] \\
&\iff (\forall n \in \mathbb{N})(\exists m \in \mathbb{N})\sim(n \leq m) \\
&\iff (\forall n \in \mathbb{N})(\exists m \in \mathbb{N})(n \not\leq m)
\end{aligned}$$

[`Typeset the following. Then decide if the statements are equivalent.`]

$$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x = -y)$$
$$(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x = -y)$$