FORENSIC ACCIDENT INVESTIGATION:
Motor Vehicles

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CHAPTER 4

Determination of Speed from Yaw Marks

Editor's Introduction.

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[Add the following at the end of the introduction:]  
During the period from 1995 to 2000—the six years following the original publication of this chapter—a great many studies of sideslip in general, and critical speed in particular, have been published. Much of this new material is critically discussed in the text supplementing this Chapter. In addition, a bibliography of the recent literature on the subject is presented, a task made easier by the several intervening papers that have themselves attempted to summarize the pertinent literature.

At the beginning of this introduction, written in 1995, it was observed that the conflict between engineering practitioners and police-investigator practitioners over the validity of the critical speed technique seemed to have been coming to an end. Six years later, I can report that there is still considerable heat and some light coming from that conflict. At the 2000 Annual Meeting of the American Academy of Forensic Sciences in Reno, Nevada, Manning delivered a provocative attack on what he viewed as the improperly broad application of the technique [Ma:00]. This stirred a spirited response by Fischer at the 2001 AAFS Annual Meeting in Seattle [Fi:01], coming at the end of a year of saber-rattling e-mails from many sources. In spite of the feeling of confrontation in the air at the Seattle AAFS meeting after Fisher completed his presentation, which included precautions that one must observe in using the critical speed approach, Manning observed that the disagreement was not as great as he and others had thought [Bo:01].
The material Dr. Semon has prepared to supplement his original chapter includes a discussion of two papers that provide a great deal of experimental information that strongly supports the use of the critical speed method, though with safeguards to ensure that the speed calculated is also "conservative" with respect to the actual speed of the vehicle that lays down the tire marks. Because of the potential importance of test data to convincing those persons who still are leery of the technique, it is essential that those carrying out the tests think through the procedures carefully. By this I mean that none of the test protocols described to date addresses the question of "blinds." Some engineers continue to be reluctant to accept the test results of police agencies, which are seen as having a vested interest in "proving" the method. Much of that reluctance could be eliminated if there were assurances that the persons who measure the yaw marks and calculate the speeds are deprived of information beforehand as to the actual speed of the vehicles laying down the marks, whether determined by radar measurement or other means. This would reduce the suspicion that the investigators were oriented toward making the calculated results lead to a certain number. I am not saying that such orientation leads to conscious skewing of the results, but that there are many unconscious ways that the skewing can occur, such as those discussed by Cheriton [Ch:97].

§ 4-2 The Basic Equations.

In summary, it is emphasized that the standard equations used to calculate the speed at which a vehicle will sideslip out of a curve have their origin in Newtonian physics. The actual sideslip is modeled by studying the motion of a "particle" traveling around a circle with a constant speed subject to the (external) forces of friction and gravity. Some have criticized this approach as being overly simplistic.\footnote{1} This attack on the theory may come from an insufficient appreciation of the theorem of Newtonian mechanics that states that the motion of any body can be considered to be made up of two independent parts:

\begin{itemize}
\item[\hspace{1cm}]\text{[Add the following to the end of the section:]}\end{itemize}
the motion of the body’s center of mass (gravity), and the motion of
the body about its center of mass. Therefore, even though the critical-
speed model used to describe sideslip situations deals with the motion
of “a particle” on a circular path, this does not mean that the vehicle
is considered to be a particle. Rather, the method is simply relying
on the fact that, in the end, it is the motion of the center of mass that
is affected by the external forces and that the incidental motion of
the other parts of the vehicle can be separated out.\textsuperscript{2}
Exactly the same approach is used to study planetary orbits. For example, if the motion
of the Earth around the Sun is analyzed, the situation is modeled as
that of a “particle” (the Earth’s mass concentrated at its center of mass)
moving around the Sun under the influence of gravity. No knowledgeable person argues that this approach is overly simplistic, in spite of
the fact, among other things, that the Earth has a non-uniform
distribution of mass. Indeed the same approach to orbital calculation
is used for really misshapen heavenly bodies, such as the oblong Eros.
\textsuperscript{1} See, e.g., [Ma:00].

\textsuperscript{2} To take a simpler example, note that if a small missile traveling parallel
to the earth strikes the left front fender of a vehicle on its side, the center
of mass of the vehicle will proceed to move (be accelerated) in the same
manner that it would had the missile somehow made a direct hit on the center
of mass itself. While it is true that the motion of the car about the center
of mass, its rotation in particular, will depend on what part of the vehicle
was struck, that motion is independent of the motion forced on the center
of mass. More concretely, if the weight of the vehicle is “W” and the impulse
(average force times the time interval that it was applied) is “J,” then the
momentum change of the car’s center of mass as a result of the missile strike
is “J = (W/g)v,” and it does not matter whether the car has been caused
to rotate by the missile or not.

Other critics of the simple theory (\textit{See, e.g.,} [SI:97], pp. 4–5) try
to “improve” upon the basic equations used in sideslip analysis by
modifying the mathematical expression for the force of friction. This
approach is inherently crippled because there is no mathematical
expression for the force of friction in the same sense that mathematical
expressions exist for the forces of gravity, electricity, magnetism, etc.
Friction is the name we give for the collective electromagnetic interactions between atoms and molecules on the boundary between two different materials, such as rubber and asphalt. To calculate an exact mathematical expression for these interactions is impossible due to the huge number of atoms and molecules involved. Hence, the force "law" for friction is an approximation, a phenomenological force law which by its nature only supports calculations to a limited degree of precision. Furthermore, in the critical-speed formulas there are other variables that have even less precision, due to measurement limitations. Therefore, there is no point in even trying to improve the description of the friction forces until there is a significant improvement in the accuracy of the other data to be used in the formula.3

In particular, it is pointless to modify the basic equation for friction so that it includes such variables as the length of the vehicle’s wheelbase, the weight distribution variation due to the amount of fuel that was on board, the cornering stiffness, etc., such as was suggested by [Di:95] until the basic data can be measured more accurately.

3 I also note in passing the continued use by [SI:97] and a few others of language such as “the critical speed is developed when the centrifugal force acting on the vehicle due to the vehicle’s motion in a circular path equals the opposing centripetal force provided by the maximum available force at the tire-road interface.” Use of “centrifugal force,” a fictitious hobbling concept, serves nothing but to add confusion to the discussion of this subject. One might just as well give as the reason your desk remains fixed in your office the fact that inertial force is holding it in place.

Consistent with the above comments, I therefore disagree with those investigators (for example, [Br:97]) who place the burden of the uncertainty in the critical-speed calculation on the uncertainty in friction. [Br:97] states that δR/R, the relative error in the determination of the radius of curvature of the yaw trajectory, is on the order of 2 percent. For the reasons I describe in § 4-3(b)(1), it is my opinion that the uncertainty in R is far greater than 2 percent if it is determined by the traditional technique involving middle-ordinate measurement. In my experience, it is the margin of error in the data used to calculate R that contributes the highest margin of error to the final estimate of...
the critical speed, not the margin of error in the measurement of friction.

It is easy to imagine that future investigators will routinely use the global positioning system (GPS) to measure the curvature of yaw marks directly, and that they also will routinely determine the frictional forces involved with sophisticated electronic instruments (such as 12-channel accelerometers, etc.). I believe that we will be using the standard formulas and methods for some time to come, and that the most important improvements in their accuracy will come, not from refining their theoretical basis, but rather from improving the accuracy of the data upon which they are based.

§ 4-3(b)(4) An Illustration of the Importance of Error Analysis.

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[Add the following at the end of the subsection:]

It is satisfying to note at this point, in early 2001, that more investigators and authors are realizing that reporting the "margin of error" in each of their measurements—as well as in their conclusions—increases our understanding of the accuracy of their speed-to-sideslip predictions and helps us better understand which factors are important in the models upon which our methods are based. Computer models currently being developed hold the potential of reducing the margin of error of the radius of curvature of yaw marks (See, for example, [Br:97], p. 244). Groups at Notre Dame University and at the University of Michigan Transportation Research Institute ([Fr:90], topic 892) are developing computer programs to simulate motion based on path-shape data obtained from electronically mapping the path. This gets away from the need to approximate a radius of curvature by the traditional means of measuring chord plus middle ordinate. Indeed, it eliminates the need to assume that the curved trajectory is an arc of a circle.
§ 4-4(a)(1) The Test Results.

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[Add the following to the end of the subsection:]

In the years since this chapter was originally prepared, a great deal of additional experimental data have become available. Some of this data had been collected earlier, and only published as part of the dispute over the critical speed method. For example, Sgt. Thomas Shelton of the California Highway Patrol (CHP) published in 1995 critical-speed-scuff data from tests carried out by the CHP and related agencies during 1979–81 and 1987–94, 94 tests in all [Sh:95]. Another data-rich paper, one reporting on 125 tests carried out in Australia over a period of several days appeared in 1997. In both instances, the discussion of the tests was aimed at supporting an operational method of obtaining vehicle speed from yaw marks for criminal prosecution purposes. The goal therefore was to have a method by which a conservative estimate of speed that was nevertheless not too far below the actual speed of the vehicle that lay the yaw marks. In other words, the goal was not to “test” the critical speed formula per se. Nevertheless, the data may lend itself to such a test.

The Australian tests will be discussed first. As part of the goal of developing an operational method, radius-of-curvature measurements were made for each test on the yaw marks left by the outside front and rear tires, respectively. Although the theory is directed at the radius of curvature of the center of mass, that is always a derived number, based on the trajectory of the yawing vehicle. Typically, police investigators implicitly assume that the center of mass is following a trajectory that is parallel to that of the tire that left the yaw mark of interest. That is, they generally determine the radius of curvature of an outside tire mark and then subtract half the car’s track width in order to arrive at the R that is substituted into the formula. That this is not always a correct thing to do can be illustrated with a yawing vehicle that has rotated about its yaw so that its longitudinal axis is far from being perpendicular to a radius vector extended from the center of curvature of the yaw trajectory.
Continuing with the attempt to develop an operational method, the study described in [Be:97] also used chords of several different lengths in determining the radius of curvature to be used in the formula: 15 m, 20 m, and (when present) 30 m.\(^1\) Finally, two different figures were used for the coefficient of friction (COF) in the speed-from-yaw formula, with the calculations all done for both the average COF and the peak COF, both determined from accelerometer measurements. As is pointed out elsewhere in this chapter and volume, the static tire/road friction is in general significantly higher than the sliding tire/road friction. Therefore, if straight-ahead braking skid tests are carried out, the deceleration at the start of the skid is higher than the average value. One speaks of the “peak” COF characterizing the start of the skid. It can be seen that if one takes the “average” COF, one is still using a higher figure than the sliding COF. Typical values of the peak COF and average COF are, respectively, 0.9 and 0.75.

\(^1\) It is presumed that the starting point for the different chords was always the same, more or less where the outside rear tire track crossed over the outside front tire mark.

There has been a continuing dispute, at least in some quarters, as to the figure that one should use as the COF in the formula for “speed from yaw.” As discussed elsewhere in this chapter, there are reasons to believe that the effective COF for a side-sliding tire is different than that for a tire that is sliding longitudinally, as in a skid to a stop. Nevertheless, because of the difficulty of measure that lateral COF, it is always the longitudinal one that is used, though with some effort to measure it in the direction that the sideslipping tire slipped. Even then, however, there is uncertainty, namely whether to use the static tire/road COF, or to use the sliding one. The argument in favor of the static COF is that, unlike the locked tire on a vehicle that is skidding to a stop, the sideslipping tire is continually introducing a new tire patch (footprint) to the pavement as the tire continues to rotate. In a sense, the tire starts slipping anew, that is, it breaks free anew continually. On the other hand, if one considers what leads a skidding tire’s COF with the pavement to be lower, one of the elements that must be considered is that the patch of tire that is in contact with
the pavement is heating up and that this may be part of what introduces effective “lubrication” between the tire and the pavement and hence the lower COF. In the case of the sideslipping tire, even though the tire patch starts to slide anew each time it is presented to the pavement, it is presented to the pavement many times a second,²² getting hotter and hotter because of the periodic sliding it experiences. All of this suggests that if it is appropriate to use the tire’s longitudinal COF, then the correct longitudinal value to use probably lies somewhere between the static COF and the sliding COF.

²² With a circumference of about six feet, the tire on a car traveling 40 mph (about 60 feet a second) will re-present the same patch to the pavement 10 times a second.

[Be:97] approached this question by doing calculations using two different values for COF: the peak longitudinal COF (which should approximate the static COF), and the average COF (which should lie somewhere between the static and the sliding values, respectively).³³

³³ A single vehicle—not one of the four test vehicles—was used to measure the COF, at least twice each day that the testing continued. As far as I could tell, they do not state how long the test skids were. In general, the longer the test skid, the lower the average COF, because the portion of the skid during which the peak COF occurs gets relatively smaller the longer the skid. It is noted in passing that the deceleration during the skid can be divided into a first, peak portion, followed by a portion during which the deceleration (and hence the COF) is lower and relatively constant. This lower value may be associated with the sliding COF. However, the implication of [Be:97] is that that is not the figure that is referred to as the “average COF.”

In addition to the experimental variables of tire pressure and vehicle, the investigation reported in [Be:97] also varied the means by which the sideslip marks were laid down. The three major⁴⁴ techniques were as follows:

(a) Driving at a constant speed in a straight direction and then suddenly steering sharply to one side or the other.⁵⁵

(b) Driving at a constant acceleration during the transition from straight motion to curved with acceleration continuing around the curved path.
(c) Driving at a constant speed in a straight direction and then suddenly steering sharply to one side or the other, with “50% braking” applied as the car rounded the curved path.

\[ \text{There were a few tests done under yet different conditions, but not in sufficient numbers to make their analysis interesting.} \]

\[ \text{The author uses the unfortunate term “oversteer” to characterize this sudden steering input, unfortunate because this term already has a widely used meaning associated with it in describing the way that a vehicle reacts to being steered. One way of describing this reaction is to say that a vehicle rounding a circular track at ever-increasing speeds must have the steering wheel turned ever “tighter” if it is in an understeer mode, must have the steering wheel “loosened” if it is in an oversteer mode, and does not have to have its steering wheel position varied at all if it is in a “neutral steer” mode.} \]

Presumably, in mode (a) the driver neither braked nor depressed the throttle, that is, the car was allowed to coast and slow down as it rounded the curve.\[ \text{There was no statement as to the rate of acceleration during mode (b). In all cases, the reference speed, that measured by radar, was taken just as the vehicle began to lay down yaw marks. The majority of the tests involved vehicles moving according to mode (a), and these are the only ones that will be discussed in any detail. In all of the 64 runs, the vehicles’ tires were inflated to the same pressure: 28 psi. In all but one of the 64 runs the calculated speed was lower than the radar-measured speed providing that the average COF was used in the calculation.} \]

\[ \text{Although this was also occasionally true if the peak COF was used, this was usually not the case. A typical one of this set of 83 runs was test 35,} \]

\[ \text{using a Ford Falcon. The radar-measured speed at the start of the curved trajectory was 100 kph. The speeds calculated using the mark of the outside front tire and the peak COF were 103, 100, and 97 kph for the 15 m, 20 m, and 30 m chords, respectively. Comparable numbers using the average COF were 94, 91, and 88 kph. Doing the calculation using the mark from the rear outside tire and the peak COF yielded speeds of 102, 101, and 98 kph, respectively, for the 15 m, 20 m, and 30 m chords. Doing the calculation with the rear-tire mark and the average COF yielded 93, 92, and 90 kph, respectively.} \]
Since the calculated speeds seemed to always or nearly always decrease the longer the chord used, it might be inferred that the speed was not being held fixed and that the car was allowed to decelerate due to the tire/road scrubbing as the yaw continued. That is, the speed calculation as a function of chord length suggests that the trajectory was spiraling inward as it progressed.

[Be:97] also describes some test runs in which the steering wheel was "flicked," with a steer to one side followed immediately by a steer to the opposite side. Although the calculated results using average COF were not as uniformly conservative with respect to the radar-measured speeds, the deviation in the positive direction remained for the most part small.

On the other hand, the very next test, test 36, was the only one out of the referenced 83 tests for which the use of the average COF yielded a calculated speed above the radar-measured speed, and it was significantly over.

I also note in passing the rather unexpected result from those tests reported in [Be:97] in which the car was being braked "50%" while rounding the trajectory in which it lay down yaw marks. In all cases, the speed calculated using the average COF value understated the speed.

Given the purpose underlying the study reported in [Be:97], it is appropriate that the method prescribed is one that deliberately underestimates somewhat the actual speed of the vehicle that lay down the yaw marks. Unfortunately, the author concludes by describing the calculated value as the "minimum" speed that the vehicle could have been going. Especially in a criminal justice situation, this is unfortunate, since "minimum" with no upper limit implies that there is no upper limit. This is an error in emphasis that also frequently occurs in the presentation of skid-to-a-stop calculations. In the case of the critical speed scuff mark calculations, it is even more important to avoid the "minimum" language. The derivation of the critical speed formula is based on the maximum centripetal force that was available from the tire/road friction. Just because one deliberately structures the calculation so as to be sure to understate the speed of the driver, it is not appropriate and is positively incorrect to characterize that
number as the “minimum.” It is simply not fair to describe something as a minimum value if one is not capable of stating an equally reasonable maximum value.

The reasoning is that the vehicle was braking between the time that the driver applied the brake and the tires began to leave marks on the road. However, rather than describing the calculated speed as a “minimum” speed, it should be described as “actually lower than the actual speed, for reasons I will explain,” or words to that effect. To do otherwise can create in the mind of the finder of fact the impression that the speed, based on the measurements made, could have been 100 mph, or 1000 mph.

The 94 tests described in [Sh:95] extended to higher speeds than are generally seen in reports of tests of the critical-speed-scruff method. The 1979–81 tests covered a range of 37 mph to 96 mph, including four above 90 mph. The tests are broken down into three categories: null (in which the vehicle was allowed to coast through the curved trajectory), acceleration, and braking. As always, the quantification of the acceleration and braking was a little weak. The braking was described as being “below lockup.” The acceleration was not characterized at all. I note in passing that one could depress the throttle enough to keep the tangential speed of the car constant and that this might be interpreted as “acceleration” by the driver. In any event, a large majority of the tests were the “null” category, 70 in all. It is likely that the number of “accelerating” and “braking” tests were too small to be significant. If this is not the case, it is remarkable that the measurements made on the yaw marks of the braking vehicles produced the largest, and indeed quite large, underestimates of the entrance speed of the vehicle. The conventional wisdom, based on the nature of the critical speed scuff phenomenon, is that the critical-speed-scruff formula will lead to an overestimate, rather than an underestimate, of the car’s speed. This is because the braking “uses up” some of the tire/road friction that would otherwise be available to effect centripetal acceleration. To the extent that the results of [Sh:95] on the braking, yawing vehicles are significant, perhaps they are due to the fact that the braking was only applied after the car was in the yaw.

This whole question as to the degree to which “starting conditions”
affect later yaw behavior is up for testing, in my opinion. This point is re-visited below in the discussion of [Br:97].

The author reports that the COF to be used in the formula was determined by using slide-to-a-stop measurements with the test vehicle at the test site and, usually, in the direction of the vehicle’s sideslip. The values of COF thus obtained were not reported. Thirteen different cars were used: a 1977 Dodge Monaco, a 1978 Dodge Monaco, a 1979 Ford Pinto, and a 1980 Dodge St. Regis in the 1977–81 tests; a 1985 Ford LTD, a 1985 Dodge Diplomat, a 1987 Dodge Diplomat, a 1985 Ford Mustang, a 1988 Ford Mustang, a 1987 Chevrolet Caprice, a 1989 Chevrolet Capric, and a 1992 Ford Crown Victoria in the later series of tests.

In the CHP tests, there was no attempt at consistency with respect to which tire mark’s radius of curvature was measured and used in the calculation. It was simply stated that the most prominent mark was used. This is usually the mark made by the outside front tire. In a few tests a series of chord lengths was used. The author states: “[a]s would be expected, the shorter chord, when measured accurately, produced the smallest error in the estimation of the test speed...” This would be better stated that the shorter chord yielded a speed closest to the speed at the beginning of the curved trajectory. The longer chords do not give an erroneous speed just because the number deviates further from the entrance speed. The vehicle is going slower and slower as it rounds the curved trajectory and taking longer chords simply incorporates more of the path where the vehicle has slowed considerably.

For the CHP tests with no acceleration or braking, the speed calculation from the critical-speed-scuff formula gave speeds that were very close to the radar-measured speeds. With only a few exceptions (and these involving underestimates) the calculated speed deviated from the radar-measured speed by less than 5 percent. To the extent that this is a solid result, it gives a big boost to the use of the critical-speed-scuff method. That said, it must be repeated that this is not a test of the theory, but the creation of a method of speed determination that yields a result that can be used in court.
It is of course always difficult to evaluate the care given to measurements like these, carried out by many different teams over many years. One would like to be reassured by a statement that the persons doing the critical-speed calculations did not have prior access, direct or indirect, to information regarding the radar-measured speeds. The pressures on an investigator to come up with an expected result, pressures of which the investigator is frequently unconscious, can skew any experiment lacking proper blinds. See, for example, "Beware Forensic Delusion," by Ross Cheriton [Ch:97]. In this context, the qualifier that the author makes regarding measurements of the radius of curvature from short chords, "if the measurement is done carefully" raises the question as to whether that measurement was repeated until the desired results were obtained and, if so, whether there was repeated measurements of the other chords as well.

Brach has reviewed the CHP data [Br:97]. He prefaced his analysis with the usual derivation of the critical-speed-scuff formula: \( v_c = [f\cdot g\cdot R] \cdot v \), and then states the conditions under which it may be reasonably used. Notable among the circumstances under which he cautions against using the critical-speed formula is that in which a vehicle has gone into a yaw because of traversing an icy path, and then lays down yaw marks once its tires again encounter a dry, high COF surface. This example may be extended to any situation in which a yaw begins on a surface with one COF and then continues onto another surface with a very different (lower or higher) COF. Will the nature of the yaw marks on the second surface continue to be affected by the motion on the first surface in such a way that an investigator just the second set of marks for a critical-speed calculation come up with a seriously wrong answer for the speed?

§ 4-4(b) Tests Done With Brakes or Accelerator Applied During a Sideslip.
by the driver of a vehicle laying down yaw marks can affect the accuracy of the speed calculation.

\[ \frac{V_{\text{calc}}}{V_{cz}} = [1 - (a_t/g\mu)]^{1/2} \]

\( V_{\text{calc}} \) is the speed that one would calculate from the radius of the yaw marks, ignoring any effect of acceleration or braking, and \( V_{cz} \) is the speed at which the car would start to sideslip given the reduced centripetal force that is available, as the result of part of the friction being "used up" by the driver’s actions in applying the throttle or brake to the car. If one plots this number versus \( a_t \), the tangential acceleration due to brake or throttle, it is found to deviate from one very slowly, thus supporting the observation that applying some braking while laying down the yaw marks does not sensibly change the radius of curvature of the marks. Nevertheless, this expression does indicate that, as it increases from zero, the calculated speed will gradually exceed the actual speed. Therefore, the reported experimental results, in which braking has actually led to the observers underestimating the speed, remain a mystery.