FORENSIC ACCIDENT INVESTIGATION: Motor Vehicles

THOMAS L. BOHAN and ARTHUR C. DAMASK, Editors
JOSEPH A. KEIERLEBER, Assistant Editor

Material to included in the 2004 Cumulative Supplement for Volume I. The insertion point references are either to the original volume or to Supplement 2003.

Chapter 1
An Introduction to the Physics of Motor Vehicle Accidents

A WordPerfect FILE IS BEING SUBMITTED SEPARATELY.

Chapter 2
Determination of Speed from Pedestrian Throw

A WordPerfect FILE IS BEING SUBMITTED SEPARATELY.

Chapter 4
Determination of Speed from Yaw Marks

[At the bottom of the material added by the 2003 Cumulative Supplement (ending on Supp page 34) to the Editor's Introduction on p. 152, add the following additional text.]

*Although more appropriately placed in a chapter dealing with basic principals, an extensive discussion of error analysis is included in Chapter 4. This discussion includes the introduction of the Monte Carlo method of analysis, which has recently been appearing in accident reconstruction reports and articles. Given the adversarial context in which the subjects presented in this book will be used, it is important to be clear about what is meant by the term**s** "error analysis," "range of uncertainty," "error bars," and the like. As Taylor [Ta:97] makes clear, the use of the word "error" in reviewing one's measurements does not mean "mistake." Also, when one talks of the range of uncertainty in one's calculated answer, one is not implying that there is some vagueness of thought.*

> In science, the word *error* does not carry the usual connotations of the terms *mistake* or *blunder*. Error in a scientific measurement means the inevitable uncertainty that attends all measurements. As such errors are not mistakes; you cannot eliminate them by being careful. The best you can hope to do is to ensure that errors are as small as reasonably possible and **to have a reliable estimate of how large they are.** [Boldface emphasis added.] [Ta:97, at 3]

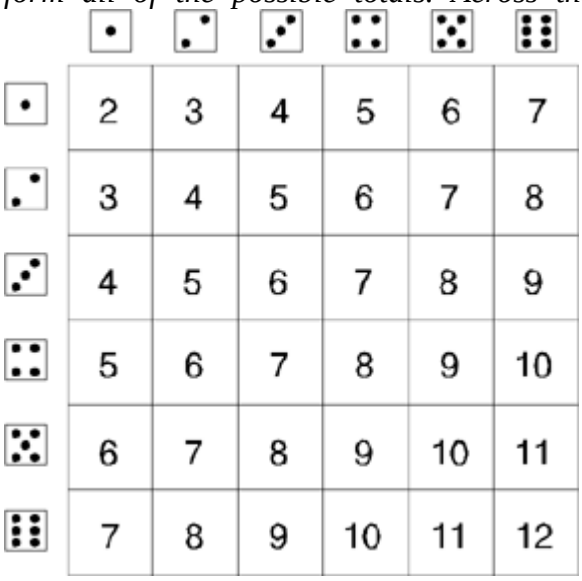
*One often speaks of random errors without being clear about what non-random errors are. In the context of making a series of measurements of some variable, a random error is an unavoidable deviation from the "true" result, one that goes in one direction as often as in the other. The statement that results of a series of measurements are valid within 3% is correct only if the **only** measurement were random. Members of the public hear or read this type of statement most frequently in the context of opinion polling. For example, tagging along after a report of the President's "approval rating" will be a caveat of the*

form "these results are valid to within plus or minus 3.2%." The correct version of this statement would be something like "If we did everything right, the results are still only valid to within plus or minus 3.2%." In that particular context, the uncertainty (error) is inherent in the enterprise of inferring the opinion of a large population from surveying a small subset of that population.

For those who visualize shooting dice better than measuring speed at the start of a skidmark, the following digression may be useful in illustrating the application of the Monte Carlo method as well as demonstrating the breadth of its applicability. Although a number of analogies exist between the rolling of dice and the measurements one makes of variables needed for a calculation, those associations will be left for the reader to make. First, consider a question that you know the answer to, or can quickly remind yourself of, namely the distribution of values one obtains by rolling two dice a large number of times. The values of each one of the two dice can be thought of as the values obtained by measuring a variable needed in an accident reconstruction calculation. In the example discussed in subsection 4-3(c)(2)(i) of the original text, there were two very different variables: skid length and speed. In rolling dice, on the other hand, the two variables are identical. The assumption that any variations (errors) in the measurement of a skid length are random and not systematic shows up in the dice discussion as the assumption that the dice are not loaded, and that only random elements in a throw determine which side of a die faces upward when it rolls to a stop. Each die is a regular cube, and each of its six sides bears a different number of pips, ranging from one to six. If the die is rolled many times, the distribution of results will be a rectangular curve, bounded by vertical lines corresponding to a minimum of one and a maximum of six, and centered halfway between three and four. The normalized height of the rectangle along a vertical axis representing probability is one sixth. (It can be seen immediately that the total area under this curve is unity, thus satisfying the requirement placed on all probability curves extending over the entire range of possible values of the variable, that the total probability of obtaining some value in the range of values is one.)

When the two dice are tossed at the same time, the possible total ranges from two ("snake eyes") to twelve ("boxcars"). The false reasoning displayed in some cross-examinations would proceed on the assumption that these and any totals in between are equally likely. It does not take a craps shooter to realize that this is not the case. Whereas there is only one combination that will result in either extreme, there are more, for some totals many more, ways of obtaining intermediate totals. Figure 4-1SE displays in matrix

form all of the possible totals. Across the top are the six values that have equal



⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Figure 4-1S Two-Dice Combinations

probability of turning up on one die; deployed vertically along the right side are the six values equally probable to show on the other die. Figure 4-2SE is a partially normalized[b] histogram of the totals which result when the dice are rolled a huge number of times. Note that a column located at a particular number along the x-axis is equal in height to the number of times that that number appears in the matrix of Figure 4-1SE. Figure 4-3SE is the same except that the vertical axis has been "normalized" so that the total area under the curve is equal to one. This figure provides the probabilities of each number being rolled on a single toss, ranging from 1/36 for either snake eyes or boxcars. The most likely roll is seven, the probability for which on a single roll is 1/6.

Since each of the two "variables" in this calculation can only take on six discrete values, the histogram depicting the totals will never become a smooth curve, regardless of how many tests are run. All that happens, as the number of tests becomes very large, is that the histogram approaches the shapes shown in Figure 4-2SE and Figure 4-3SE. Figures like these will appear in the Monte Carlo calculation, where a bell-shaped distribution usually replaces the uniform[c] distribution found in the case of the dice. The following steps set out how a Monte Carlo method is applied in the case of the dice:

1. Using a random number generator, generate a value for die 1 between one and six, X1.
2. Using the random number generator, generate a value for die 2 between one and six, X2.
3. Add X1 and X2, placing the sum on a histogram array.
4. Repeat steps (1), (2) and (3) N times.

---

[b] Think of it as the result of 36,000 rolls of the dice, but where each total has been divided by a thousand; or of 36,000,000 rolls where each total has been divided by a million.

[c] By "uniform," it is meant that the probability is the same for throwing any value on the die.

This is in effect the procedure followed by the Monte Carlo method once one has provided the shape of the uncertainty distributions of the underlying variables (i.e., of each die (as described in footnote (c)).

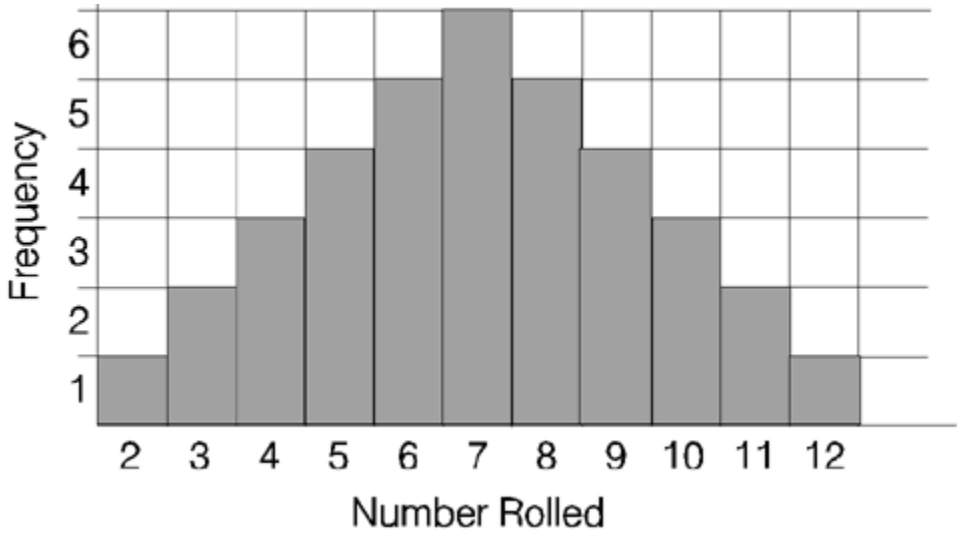


Figure 4-2SE Relative Frequency

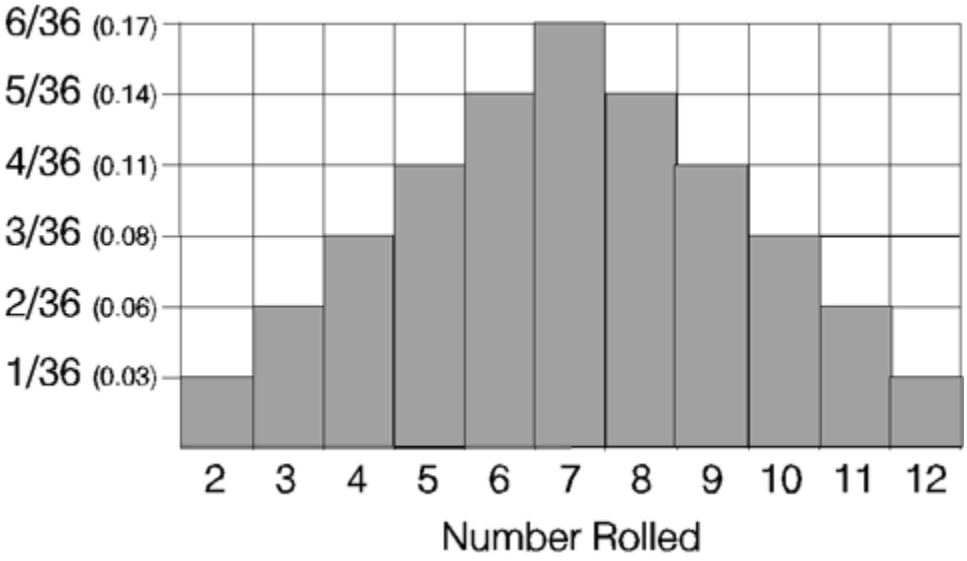


Figure 4-3SE Normalized Frequency (Probability)

[Replace the present subsection §4-3(c)(5) with the following material, renumbering the existing subsection §4-3(c)(5) as §4-3(c)(6).]

§ 4-3(c)(5). Further Discussion of Uncertainty; Introduction to the Monte Carlo method.

Section 4-3(a) emphasized the importance of including a margin of error when presenting any number. However, it was not very explicit about what "margin of error" meant. In fact, the phrases "margin of error," "degree of uncertainty,"[d] and variants on them get tossed around a good deal by scientists, engineers, and lawyers in speaking with one another under circumstances where it is likely that they each have a different meaning – including *no* meaning – in mind. In the context of scientific measurement, "error" does not refer to a defect in the measurement or a mistake in technique, but to the inescapable uncertainty that results even when the measurement is done perfectly. "Error" and "uncertainty," which have the same meaning in this context, arise in discussions aimed at defining the precision with which we know a particular quantity, whether it was measured *directly,* or calculated from two or more quantities that had been directly measured.

This section will discuss a particularly powerful approach for accomplishing an uncertainty analysis, an approach that has only within the last decade or so appeared in the field of motor vehicle accident investigation. It is hoped that this discussion will provide the reader[e] with sufficient understanding to be able to ask—and respond to—intelligent questions regarding margin of error in general, and to be able to efficiently dispatch non-intelligent questions and answers on the subject.

To illustrate one particular Monte Carlo method, the "rejection Monte Carlo method," a somewhat artificial accident-reconstruction-related problem will be used: the skid-to-a-stop determination of μ, the coefficient of friction (COF)**,** between sliding tires and pavement. Here, after two variables are physically measured, μ is calculated from an expression containing those variables. The variables to be measured are the length of the skidmark (x) and the speed of the test car at the start of the skidmark (v). In general, there will be a range of uncertainty in the results of both measurements. As discussed earlier in this section, the margin of error that is of ultimate interest is that in the calculated quantity. This is sometimes referred to as the **"**composite error,**"** and is determined by the individual margins of error of the underlying measurements.

In the skid-to-a-stop method of finding the COF between tires and a particular road surface (a method which has been used relatively infrequently in recent years), a test car's brakes are slammed on while the car is traveling over the surface of interest at a measured speed. To spare the tires of the test vehicle, that speed was typically no higher

---

[d] Labeling from the opposite perspective is plagued by an equal number of terms, such as "degree of certainty," "confidence level," etc., most of which are also mutually interchangeable.

[e] As always, the reader is presumed to be a member of the legal profession, or a technically trained and experienced individual engaged in forensic work. It is also presumed that the technical ability within both subsets has a considerable range, even though the bottom of that range will be higher with the forensic workers than with the attorneys. The material has been developed with the intention that, even if some parts are difficult going for some of the readers and other parts overly simplified for other readers, all readers will find parts of the material useful.

than about 30 mph. However, for our example, we will select $v = 60$ mph as the measured speed at the instant the first skidmark begins.[f] That is, the brakes are slammed on and the test car skids to a stop. We specify further that the skid occurs on a horizontal surface, that the front tires lock up first and that the length of the longer of the two front-tire skidmarks is measured to be $x = 180$ ft. Using Eq. (4-11) we find

$$\mu = \frac{v^2}{2gx} = \frac{88^2}{(2)(32.2)(180)} = 0.67.$$

Actually, we have already made a statement about the precision with which μ is known, since 0.67, with its two significant figures is obtained from rounding off the longer number that results in carrying out the above calculation: 0.66804… This rounding off, in deference to the speed being known to no better than two significant figures, should be second nature.[g] But this is just the beginning of error analysis. After the uncertainty in the underlying measurements is determined, it may be that even this degree of precision will be found to be absent.

We now can calculate the coefficient of friction μ and its associated margin of error $\Delta\mu$ using the Monte Carlo method which, though used successfully in nuclear physics since 1944[h], has only recently been applied to calculations in accident reconstruction. (For two of the earliest papers applying Monte Carlo to accident reconstruction calculations, see Ko:94 and Wo:94.)

The Monte Carlo method has three main advantages over the two methods discussed in Section 4-3(a):

---

[f] Note that this (somewhat artificial) approach avoids the huge source of error discussed in Section 4-3c(2)(i), an error resulting from the fact that a car can travel a significant distance between the instant the brakes are applied and the instant the tires start laying down skidmarks. In such circumstances, a ridiculously high value is calculated for the COF, regardless of how precise the speed at braking and the length of the skid are measured. An error of this nature is called a "systematic error," since it systematically (always) errors on one side of the true value.

[g] Most reconstructionists seem to know this rule when it comes to the COF. The situation is vastly different when it comes to calculated speeds, which are often seen stated to four, five, or six significant figures. Probably the difference arises because of the way the COF is listed in the literature: always with two-significant-figure precision, and never with three or more.

[h] The idea of modeling complex phenomenon by making various trials and studying the outcomes occurred to Stanislaw Ulam while he was playing solitaire during an illness in 1946 ([As:89 on 312). The immediate problem he was trying to solve was that of predicting the results of neutron diffusion in fission experiments at Los Alamos. The name of the procedure first appeared in the paper by Metropolis and Ulam in 1949 [Me:49]. The first simple algorithm to set up a Monte Carlo method was the Metropolis algorithm introduced by Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller in 1953 [Me:53].

first, it provides a way to express margins of error in language that is easy to understand and difficult for an adversary to distort;

second, it provides an easy way to calculate the composite margin of error in a calculated quantity even when the mathematical expression from which it is obtained is relatively complex, such as is the case for v_{cs} (Eq. (4.4)); and

third, it provides a way to include information in the calculation of the margin of error that otherwise could not have been used.

In sum, not only does the Monte Carlo method provide an accurate expression for the composite margin of error, it also provides one that can be understood intuitively.

Before getting into the Monte Carlo method *per se* it is useful to discuss a few margin-of-error basics. It is general knowledge that in the absence of outright mistakes, the more times one measures a quantity of interest the more precisely that quantity can be expressed. Thus, in the example above, suppose that we measure the skid distance many times. Earlier, it was simply stated to be 180 feet, which may be the result of a single measurement. Making many careful measurements, however, will not result in a single number for the length, but rather in a distribution of values centered on a particular value. This distribution arises for a variety of reasons, even in the absence of carelessness or faulty technique. For example, the people making the measurement may be inconstant in the degree to which the measuring tape is held taut. Also, given the faintness with which most skidmarks are characterized at their starting point, the end of the measuring tape may not be laid down at exactly the same point each time. The idea of random error is that it subtracts from the "true" value as often as it adds to it and so in that sense averages out.

Imagine that the skidmark length measurement has been carried out twenty times with the results as plotted in the histogram of Figure 4-4S, which shows the number of times each value was obtained during those twenty measurements. In other words, the histogram is a plot of the measured length (along the horizontal axis) versus the number of times that value was obtained (vertical axis). It can be shown that if the measurements are indeed subject only to random errors, the histogram will eventually look something like Figure 4-5S after many (e.g., 1000) measurements have been made. (See, for example, Br:94, Ba:02, or Chapter 5 of Ta:97.)

We next "normalize" the histogram of Figure 4-5S by dividing the units on the vertical axis by the total number of measurements made of x, obtaining Figure 4-6S. Once this is done, each vertical column has a height equal to the *fraction* of the total measurements that resulted in value or range for x at the base of that column. For example, if 300 of 1000 measurements resulted in a value of x between 176 and 178 ft, then the height of the column above the interval from 176 to 178 in Figure 4-6S, the normalized histogram, would be 0.30 (= 300/1000), as opposed to 300 on the non-normalized Figure 4-5S. Of course the two graphs have exactly the same shape, since only the scale on the vertical axis has been changed.

As the number of measurements increases, a smooth curve of the shape referred to as a "bell curve" begins to emerge from the irregular "staircase" of the initial histogram, as shown in Figure 4-6S. Another name for the bell curve is "Gaussian distribution." A histogram representing multiple measurements of any quantity often will approach the shape of a Gaussian distribution as the number of measurements becomes large, as long as the measurement process producing the numbers is subject only to *random* errors.[i] Thanks to the work by Gauss**,** which led to his name being applied to this distribution, the properties of the Gaussian distribution[j] are known in great quantitative detail. Among other things, the probability that the true value for the measured entity lies within a particular range can be quantified. For example, once we have enough measurements of *x*, we can state the probability that, instead of being 180 feet, the true value of *x* is 170 feet, or 191 feet, etc.

There are many approaches to defining the uncertainty in a measurement. One often sees the COF written in the form $\mu = 0.8 \pm 0.1$, which looks impressive (in terms of recognizing uncertainty) but is undefined without a statement as to what the error limits represent. As discussed in Chapter 2 in the Supplement, the choice one makes in defining the error limits depends on the context. Although making this assumption in non-scientific fields is risky, it is often assumed that, in the absence of an explicit definition of the error limits**,** those limits represent the standard deviation of the Gaussian distribution of results. (Clearly, this also involves an assumption that the results follow a Gaussian distribution.) Unfortunately, in the courtroom, it is often assumed that error limits define some kind of "uniform" range within which the quantity in question has an equal probability of lying. This is essentially always wrong.[k]

Most textbooks on probability (see, in particular, Chapter 5 of the very readable book by J.R. Taylor [Ta:97]) show that the center of the Gaussian distribution is the average value of the results from a great many measurements of the particular variable.[l] If most of the measured values lie close to the average value, the distribution then is said to be "well-peaked," as shown in Figure 4-7S. It is the distribution's standard deviation (just mentioned) that characterizes the tightness of the data around the average, or

---

[i] Repeating the example cited in the Editor's Introduction (Supplement), it is noted that this assumption of no non-random errors underlies statements such as "the margin of error in this survey was 3.2%," usually made nowadays after describing the results of an opinion poll. A less misleading statement would be something like "Assuming everything was done correctly in carrying out the survey, these results have a margin of error of 3.2%." Or, alternatively, "The absolute minimum margin of error for this survey is 3.2%; it may be much larger."

[j] A Gaussian distribution is also referred to as a Gaussian curve or, for short, simply, a Gaussian.

[k] One example of where this "square" range *is* correct is in the roll of a single die. If the die is not "loaded," it has an equal probability of having any one of its six sides facing up at the end of the roll.

[l] Any quantity substituted into an expression to be evaluated is referred to as a variable, and generally as an independent variable. This does not suggest that there is any variability to the quantity in a given context. For example, the skidmark length is what it is. However, the next time, it will be different even though the same expression will be used to calculate the COF. In this context the COF is the dependent variable, because it depends, through Eq. 4-11 on the two independent variables: *x* (the skidmark length) and ***v*** (the starting speed).

alternatively, the spread of the measured data about the average value. The standard deviation will have the same units as the variable being measured, and is traditionally represented by the lower case Greek letter sigma: σ. Figure 4-8S shows one particular σ. Stated differently, a small σ means there is a very small probability that the actual value for the measured quantity differs significantly from the average, which is what is really meant when one says that there is only a small margin of error associated with the measurement. Equivalently, one can say that a small σ means that the uncertainty of the variable is small.[m] Conversely, a large σ means there is a large margin of error, that a single measurement of the variable has a significant chance of deviating significantly from the true value, and that, after many measurements, there remains a significant chance that the true value deviates significantly from the average. The Gaussian associated with a particular variable is in fact the probability that that variable's true value lies at any particular interval along the x-axis. This means that we can do away with vague descriptions about the probability of a particular error. For example, for a variable *x* having a Gaussian probability distribution characterized by an average value x_{av} and a standard deviation σ, the probability is 68% that a random measurement of *x* will yield a value falling within the range $x_{av} \pm \sigma$.[n] Similarly, there is a 95.4% probability that any measured value of x lies in the range $x_{av} \pm 2\sigma$, a 99.7% probability that any measured value of x lies in the range $x_{av} \pm 3\sigma$, etc. Indeed, as illustrated in Table 4-1S, it is easy to find the probability that any measured value of x will be in the range $x_{av} \pm t\sigma$, for any value of *t*.

We can now describe the Rejection Monte Carlo method in the context of the calculation of the margin of error for COF as determined by the skid-to-a-stop method. In this case, there are two measured quantities, x and *v*, from which we will calculate the coefficient of friction μ and its associated margin of error $\Delta\mu$. The steps below are illustrated in Figures 4-8S and 4-9S for one variable, the distance *x*:

1) Assume the measured quantities x and *v* follow normalized Gaussian distributions centered at x_{av} and v_{av}, with standard deviations σ_x and σ_v. One may choose to characterize each margin of error as *being* σ, so that one would state the result of the measurement of, say, the skid distance, as $^{o}x = x_{av} \pm \sigma_v$. Alternatively, rather than use each σ to define each margin of error, one may look to the full range of values obtained for each quantity so, for example, $(x_{\max}, x_{\min})$ would determine the

[m] Going just a bit further, one can say that if multiple measurements of a particular quantity always show a very small range of values (a small scatter) then it is likely that each measurement of this quantity has a high degree of precision.

[n] It is noted that $x_{av} \pm 0.67\sigma_x$ defines the 50% range. That is, there is a 0.50 probability that the true value of *x* lies in this interval. It follows that the probability that the true value lies outside that interval is also 0.50.

[o]

beginning and end of the distribution. If the data is symmetric in the sense that x_{max} is the same distance *above* x_{av} as x_{min} is below it, then one again has a simple expression for $x = x_{av} \pm \delta$, where δ is the distance from x_{av} to each of the extremes. (Cases in which measured data do not follow Gaussian distributions are discussed below.)

2) Use a random number generator[p] to obtain a value of x within some defined range, such as $x_{min} \leq x \leq x_{max}$, or $(x_{av} - 3\sigma) \leq x \leq (x_{av} + 3\sigma)$, etc.

3) Generate a second random number whose value lies between zero and one. This corresponds to picking a particular value on the vertical axis of the normalized distribution of measurements of x.

4) Determine whether the random number generated in step (3) is greater than or less than the value of the normalized Gaussian at the random value for x generated in step (2).

5) Repeat steps (2) through (4) with the second measured quantity v; i.e. repeat steps (2) through (4) substituting v for μ.

6) (a) If step (4) results in a "greater than" finding for either variable then discard both values and start again at step (2).

(b) If step (4) results in a "less than" finding for *both* variables, use the values of x and v thus obtained to calculate μ, and record that value on a μ-histogram.

7) Repeat steps (2) through (6) many times. This will produce N values for μ. (Note that N will be less than the total number of runs because of the "rejection" condition of step (6)).

8) Normalize the complete μ-histogram by dividing the height of each column by N.

After enough loops through the calculation, the μ-histogram will approach a Gaussian distribution, yielding an average value for μ and a standard deviation σ_μ. This information can then be used to state the desired result: $\mu = <\mu> \pm \sigma_\mu$.

---

[p] Monte Carlo software packages are available for both stand-alone use or for use in conjunction with widely used spreadsheet applications. These packages can handle the random-number generation as well as the other calculations needed in the application of the Monte Carlo method.

The expression obtained for μ and its margin of error in the final step above is the result sought. The margin of error thus obtained is essentially the same as that calculated by the most accurate method of error analysis, the method of quadratures [Br:94 and Chapter 3 of Ta:97]. However, the Monte Carlo method run on a modern microcomputer is faster and less tedious. This convenience in especially pronounced in those instances where three or more measured variables are required for the calculation.

Even in this simple example, it's easy to see how valuable the results of the Monte Carlo method can be in testimony and depositions. For instance, suppose an attorney says "So, Doctor Professor X, the value of the coefficient of friction could be as low as 0.6 [naming the very bottom of the uncertainty range]." If the expression for the COF has been obtained with the Monte Carlo method,[q] the witness can respond with something like "Yes it *could* have been that low; however, in 200,000 computer simulations this value occurred only1000 times, or less than 1% of the time." Testifying in this way eliminates the distortions that can occur when a an adverse witness or attorney picks up on a world such as "uncertainty" in order to suggest to a judge or jury that the whole calculation is "uncertain."

A second (and perhaps unexpected) advantage of the Monte Carlo method is that it allows considerations or knowledge to be incorporated that otherwise would not play a role in the calculations. [Wo:94, Mo:03 on p. 44] For example, suppose an automobile started from rest a short distance before it was involved in a collision. Knowing this puts a limit on the speed it could have had at the place of impact. This limit is easily incorporated into the Monte Carlo method by using it as an additional acceptance/rejection criterion in step (6). Thus, we could not only require the random value used in the rejection/acceptance step to be less than the Gaussian representing the distribution of speed, we also could require that value to be less than the maximum speed the car could have obtained by virtue of starting from rest. Criteria from other considerations, such as (possibly) witness testimony, internal consistency checks, crush damage, momentum or energy considerations, etc., can be incorporated in the same way, as additional requirements a measured variable has to satisfy in order to be used in a Monte Carlo run. In this way, restrictions which otherwise would not have played a role in the calculation of μ can lead to a more accurate answer with a smaller margin of error than would have resulted from using only the measured quantities, *and* with a traceable means of showing what restrictions were imposed. Obviously, series of runs with and without any one of the restrictions can be displayed.

The next advantage of the Monte Carlo method has to do with the way in which it handles the probability distribution for each measured value. In the example above we assumed the values of both the speed and the COF followed a Gaussian distribution. However, we have the freedom to choose any distribution we want, as well as a different distribution for each variable. For example, in cases where a distribution is unknown, some investigators use the "uniform distribution," in which all values within a certain

[q] Even if it is a direct measurement, by a means known to produce a Gaussian distribution, the witness can intelligibly characterize the probability that the true value is at the bottom of the stated error range.

range are assumed to be equally probable.[r] [Ba:03, Wo:94, Mo:03] Kost and Werner [Ko:94] have investigated the effects of using three different types of distributions: the Gaussian distribution, the uniform distribution, and the triangle distribution (which has a maximum at the average value, and falls linearly to zero at the two limits to the stated uncertainty range.). Other considerations involved in choosing a distribution are discussed by Moser et. al. [Mo:03]

Important information about the actual distribution of quantities frequently measured in accident reconstructions has been obtained by Goude et. al. [Mo:03 on p. 47], and especially Bartlett et. al. [Ba:02] They all found that measurements of a COF (by other than by using a drag-sled[s]) do follow a Gaussian distribution, as do measurements of the length of skidmarks and the radius of curved tire marks. On the other hand, their data calls into question the use of a Gaussian distribution for estimates of crush damage, and for measurements of friction coefficients with drag sleds

The actual running of a Monte Carlo analysis can be done using a commercial program, such as Crystal Ball [Ko:94], or can be carried out with a computer program written by the individual investigator, as was described in [Ba:03] and [Mo:03]. The individual following the latter course must take care of a number of factors, such as the choice of a random-number generator, the number of Monte Carlo runs needed for convergence to a Gaussian, the units to be used on the horizontal axes, etc.

A number of interesting examples of the Monte Carlo method have been presented in the literature. Bartlett [Ba:03] used it to analyze the stopping distance of a motorcycle with rear-wheel-only braking, and a problem involving the distance required to stop when perception/reaction time is taken into account. Moser et. al. [Mo:03] analyzed a ninety-degree collision; Wood and Riordain [Wo:94] analyzed two relatively complex collisions, a rear end collision and another collision in which the location of the point of collision was unknown. Kost and Werner [Ko:94] used the method in another complex example, in which a car wandered off the road onto the right shoulder, then crossed the road to the left side as a result of an over-correction of the driver, after which it left the roadway surface and traveled on a grassy shoulder. The Monte Carlo method allowed them to use different probability distributions in different parts of the analysis.
In summary, the main advantages of using the Monte Carlo method for calculating margins of error are:

1) The results are easier to present and explain in a courtroom or deposition, and are less subject to distortion.

---

[r] This is also the assumption made by the attorney probing how they may use the uncertainty range to get the final result as favorable to his client as possible. However, it is one thing to do this with a single measured quantity and quite another to do it with a composite uncertainty range.

[s] It was found that unless all the measurements of a particular COF were performed with the same technique and a high quality spring scale they did not form a Gaussian distribution. This should come as no surprise to those who have tried to use a drag sled for this measurement. In addition to all the other problems inherent in this approach, the low speed of the measurement results in the reading on the scale jumping all around. One cannot help but be dubious of those drag-sled-based COF-measurement reports wherein each pull of the drag sled is said to require a well-defined horizontal force precise to two significant figures. The reason that not more is made of this questionable practice is probably the fact that the end values of COF are reasonable and the final results (e.g., calculated speed) are only weakly affected by errors in the COF.

2) The method can be successfully applied to cases in which the equations used are relatively complex, or in which several different equations must be used to get the final result.
3) The method can successfully incorporate other considerations, such as witness testimony, limits on variables from other conditions, etc., to get a better estimate of the final result and reduce its associated margin of error.
4) The method allows different probability distributions to be used for different variables.

As:89 W. Aspray, in *The History of Modern Mathematics, Vol. II: Institutions and Applications*, ed. D.E. Rowe and J. McCleary, Academic Press, Boston, 1989. [4]

Ba:02 W. Bartlett, W. Wright, O. Masory, R. Brach, A. Baxter, B. Schmidt, F. Navin, and T. Stanard, "Evaluating the Uncertainty in Various Measurement Tasks Common to Accident Reconstruction," SAE Technical Paper 2002-01-0546, Society of Automotive Engineers, Inc., Warrendale PA, 2002. [4]

Ba:03 W. Bartlett, "Conducting Monte Carlo Analysis with Spreadsheet Programs," SAE Technical Paper 2003-01-0487, Society of Automotive Engineers, Inc., Warrendale PA, 2003. [4]

Br:94 R. M. Brach, "Uncertainty in Accident Reconstruction Calculations," SAE Technical Paper 940722, Society of Automotive Engineers, Inc., 1994. [4]

Ko:94 G. Kost and S. Werner, "Use of Monte Carlo Simulation Techniques in Accident Reconstruction," SAE Technical Paper 940719, Society of Automotive Engineers, Inc., Warrendale PA, 1994. [4]

Me:49 N. Metropolis and S. Ulam, *J. Am. Stat. Assoc.* Vol. 44, p. 335, 1949. [4]

Me:53 N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, and E. Teller, *J. Chem. Phys.* 21, 1087, 1953. [4]

Mo:03 A. Moser, et. al., "Application of the Monte Carlo methods for Stability Analysis within the Accident Reconstruction Software PC-CRASH," SAE Technical Paper 2003-01-0488, 2003. [4]

Wo:94 D. P. Wood and S O'Riordain, "Monte Carlo Simulation methods Applied to Accident reconstruction and Avoidance Analysis," SAE Technical Report 940720, Society of Automotive Engineers Inc., Warrendale PA, 1994. [4]

Ta:97 J.R. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, 2nd Ed., University Science Books, Sausalito, CA, 1997. [4]