Math 206 Section A
Review for Test 2

The test is on **Friday, October 12** during our class time. The test will cover sections **3.1-3.6, 4.1, 4.2, 4.4, 5.1, 5.2**.

1. Sketch the following sets. Determine if the sets are open, closed, or neither. Use the definition of open sets and closed sets to illustrate this with your sketch. Also find the boundary and complement.

   (a) $A = \{(x, y) \mid 4 \leq x^2 + y^2 < 9\}$

   (b) $B = \{(x, y, z) \mid y \geq 1\}$

   (c) $C = \{(x, y, z) \mid \|(x, y, z) - (1, 1, 1)\| < 2\}$

2. Sketch the set $B_2(1, 2)$ in $\mathbb{R}^2$.

3. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + y^4}$.

4. Determine the points of discontinuity in the given function. Are the discontinuities removable?

   (a) $f(x, y) = \dfrac{x^3 + x^2 + xy^2 + y^2}{x^2 + y^2}$

   (b) $g(x, y, z) = \dfrac{3x^4 - y^5 + z^6}{x^4 + y^4 + z^4}$

   (c) $f(x, y) = \dfrac{\ln(1 - x^2 - y^2) + x^2 + y^2}{x^2 + y^2}$

   (d) $h(x, y, z) = \dfrac{x^3 + x^2 + xy^2 + y^2}{x^2 + y^2}$

5. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} \dfrac{3x^2y - y^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

   Find the first partial derivatives of f with respect to each independent variable. (To find the partial derivatives at the point $(0, 0)$, you need to use the limit definition of the partial derivative.)

6. Suppose that a bird flies along the helical curve $x = 2\cos t$, $y = 2\sin t$, $z = 3t$. The bird suddenly encounters a weather front so that the barometric pressure is varying rather wildly from point to point as $P(x, y, z) = \dfrac{6x^2z}{y}$ atm.

   (a) Use the chain rule to determine how the pressure is changing at $t = \pi/4$ min.

(b) What is the approximate pressure at $t = \pi/4 + 0.01$ min?

7. Let $f(x, y) = (x^2 - y^2, 2xy)$ and $\vec{a} = (1, 0)$.

   (a) Write a formula for $Df(\vec{a})$.

   (b) Describe in words the action of the linear transformation $Df(\vec{a})$ on vectors in $\mathbb{R}^2$.

   (c) Use $f(1, 0)$ and $Df(1, 0)$ to find an approximation for $f(0.99, 0.01)$.

8. An unevenly heated plate has temperature $T(x, y)$ in °C at the point (x, y). At the point $(2, 1)$, the temperature increases at the rate of 16°C/m in the x-direction and it decreases at the rate of 15°C/m in the y-direction.

   (a) Find the differential dT at the point $(2, 1)$.

   (b) Estimate the temperature at the point $(2.04, 0.97)$ if $T(2, 1) = 135$.

9. Show that $T(x, y, t) = e^{-kt}(\cos x + \cos y)$ satisfies the two-dimensional heat equation

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial T}{\partial t}.$$

10. Suppose the function $f(x, y)$ gives temperature in °C at the point (x, y) and x and y are in cm. The function f is differentiable and $f_x(1, 2) = -3$, $f_y(1, 2) = 4$ and $f(1, 2) = 7$.

    (a) Describe in words the meaning of the statements $f(1, 2) = 7$ and $f_x(1, 2) = -3$.

    (b) A bug leaves the point $(1, 2)$ to cool off as fast as possible. In which direction should the bug head?

    (c) Find a vector perpendicular to the level curve of f at the point $(1, 2)$ and explain what the vector tells you in terms of temeperature.

    (d) Find an equation for the tangent line to the level curve of f at the point $(1, 2)$.

11. Find $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = -y\vec{i} + x\vec{j} + 5\vec{k}$ and C is the helix that starts at the point $(1, 0, 0)$ and winds counterclockwise around the z-axis twice.

The following problems are from the textbook.

12. Page 160: Problem 11 (After you sketch the level curves, draw the gradient vector at the point $(1, 1)$ on the same sketch. Do not compute the vector, but simply draw it by looking at the level curves. Your drawing will be an estimate.)

13. Page 161: Problem 23 (After you sketch the vector field, draw a curve C such that $\int_C \vec{f} \cdot d\vec{r}$ is negative.)

14. 17 (page 185); 25, 29 (page 209); 25, 41 (page 223); 5, 11, 28 (pages 229-231); 19, 21, 29, 43 (pages 249-251); 3, 5, 7, 13, 15, 19, 27 (pages 292-294).