

## Basic Functions and Their Inverses

**Definition.** A function is a rule that assigns to every  $x$  value in the domain, one and only one  $y$  value in the range.

**Definition.** A function is **one-to-one** if for every  $y$  value in the range, there is one and only one  $x$  value such that  $f(x) = y$ .

**Definition. Inverse Function:** Suppose  $f(x)$  is a one-to-one function with domain  $D$  and range  $R$ . The inverse function  $f^{-1}(x)$  is defined by

$$f^{-1}(b) = a \text{ if } f(a) = b$$

The domain of  $f^{-1}(x)$  is  $R$  and the range of  $f^{-1}(x)$  is  $D$ .

**Finding an Inverse:**  $f^{-1}(x)$  is a reflection of  $f(x)$  through the line  $y = x$ . To calculate  $f^{-1}(x)$ :

1. Solve the equation  $y = f(x)$  for  $x$ . This gives a formula  $x = f^{-1}(y)$  where  $x$  is expressed as a function of  $y$ .
2. Interchange  $x$  and  $y$  to obtain the expression  $y = f^{-1}(x)$

### Some Standard Functions

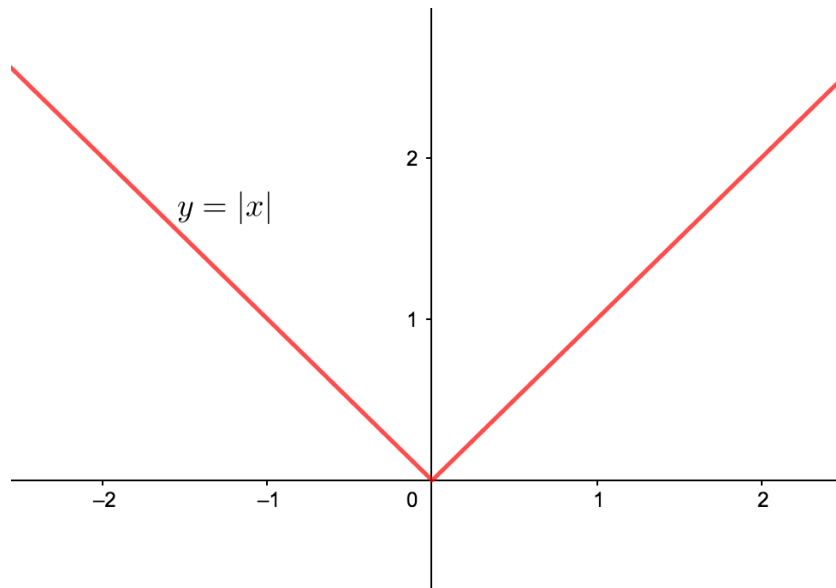


Figure 1: The Absolute Value Function

Note: This function is NOT one-to-one so it does not have an inverse. However, you could invert the section where  $x \geq 0$  or the section  $x \leq 0$

Some Standard Functions and Their Inverses

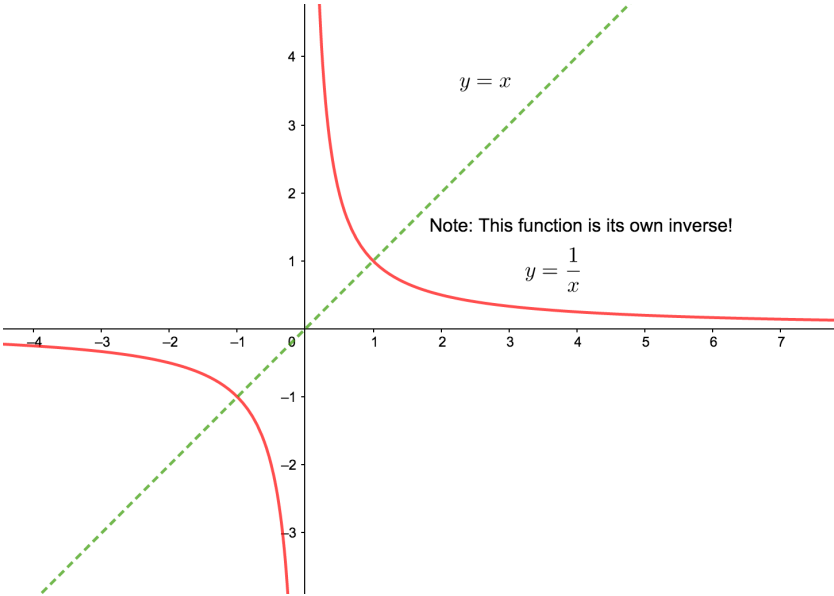


Figure 2: The Reciprocal Function

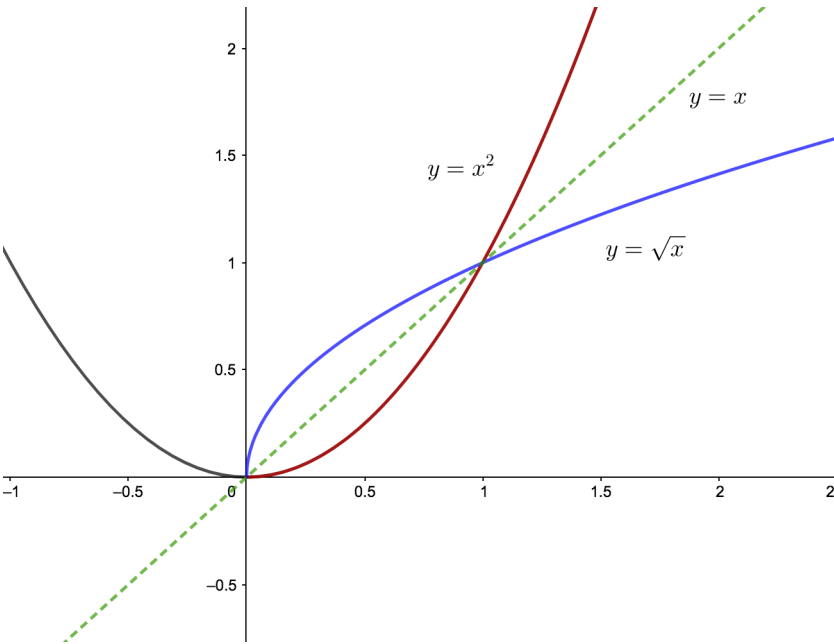


Figure 3: The Quadratic Function and its Inverse

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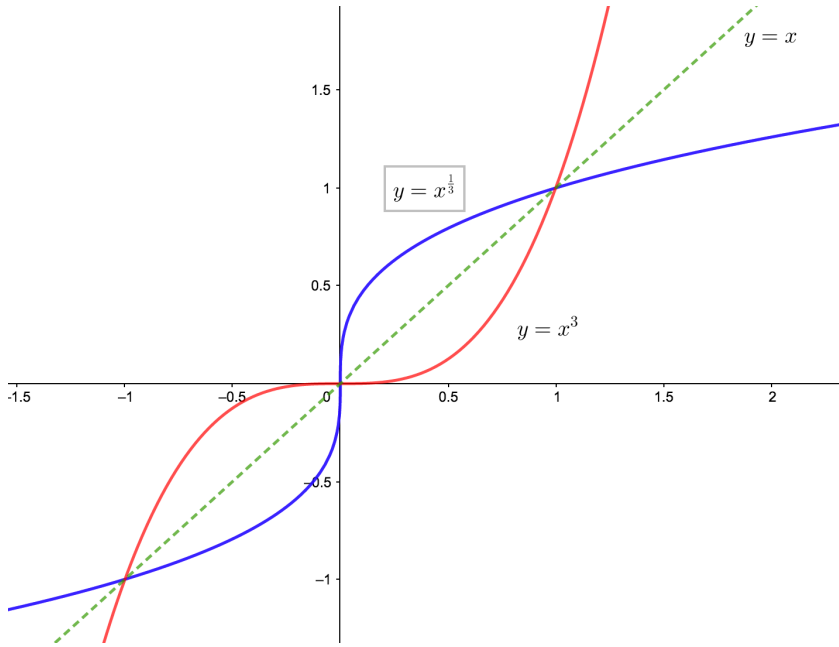


Figure 4: The Cubic Function and its Inverse

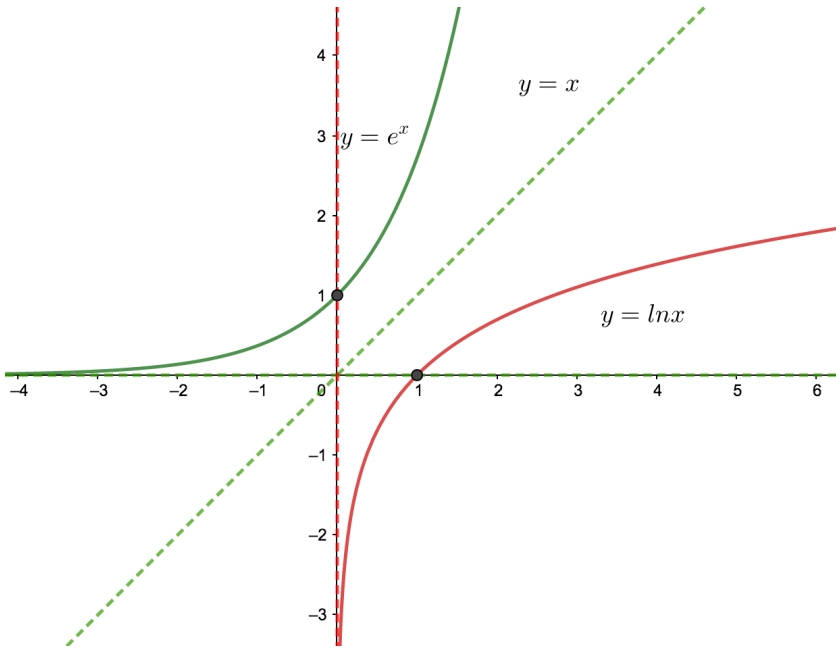


Figure 5: The Exponential Function and the Natural Log

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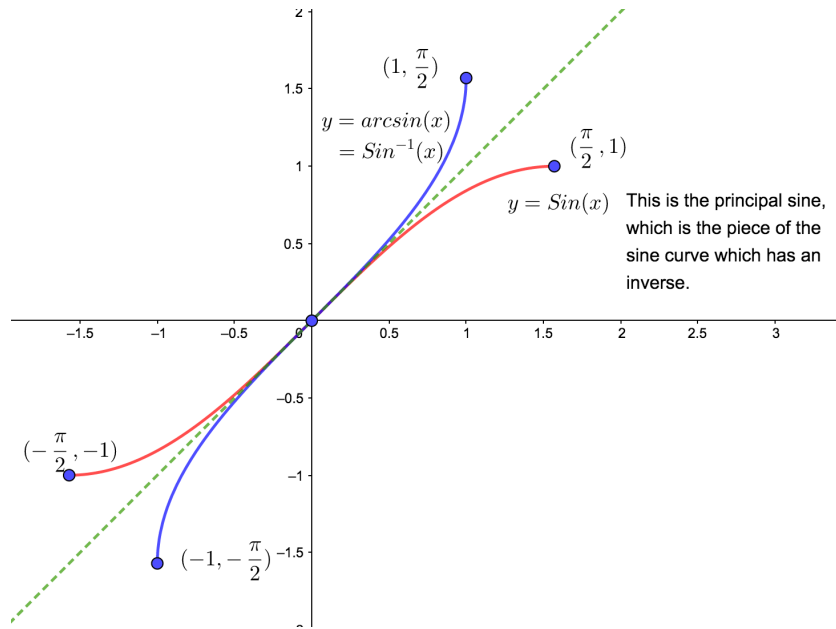


Figure 6: The Sine Function and its Inverse

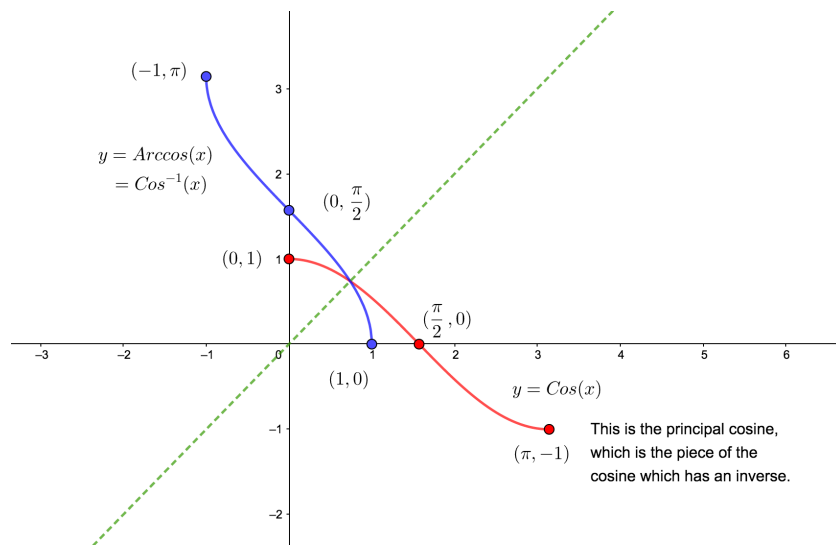


Figure 7: The Cosine Function and its Inverse

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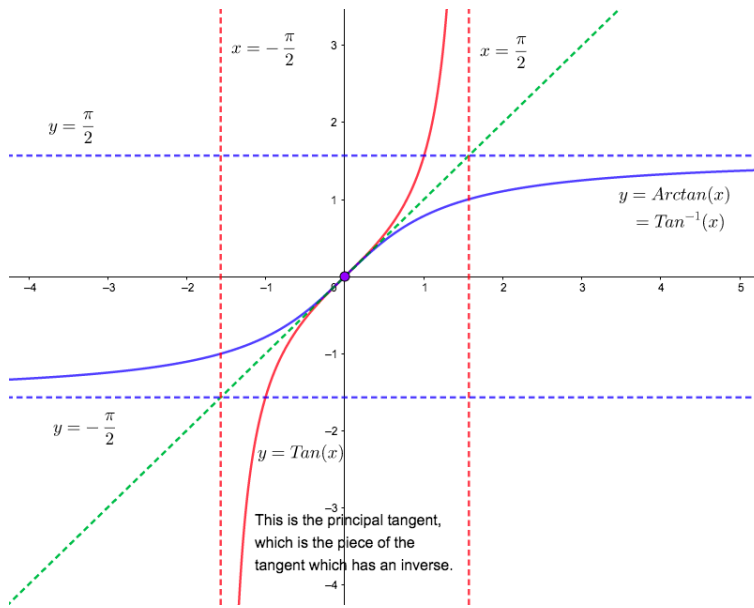


Figure 8: The Tangent Function and its Inverse

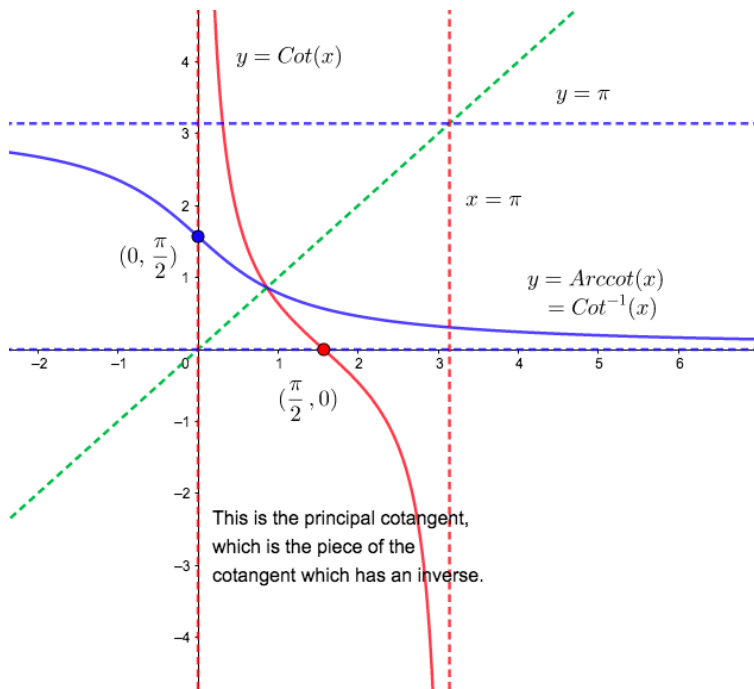


Figure 9: The Cotangent Function and its Inverse

## Some Standard Functions and Their Inverses

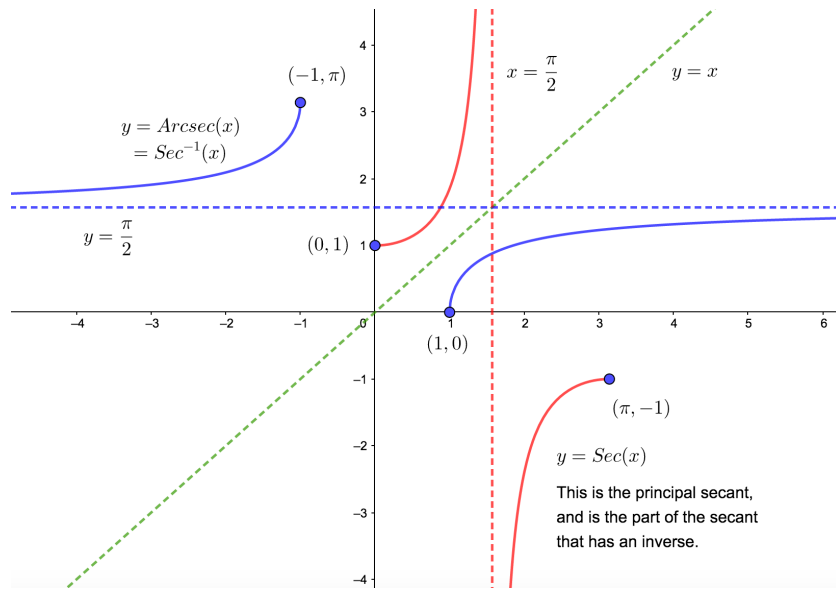


Figure 10: The Secant Function and its Inverse

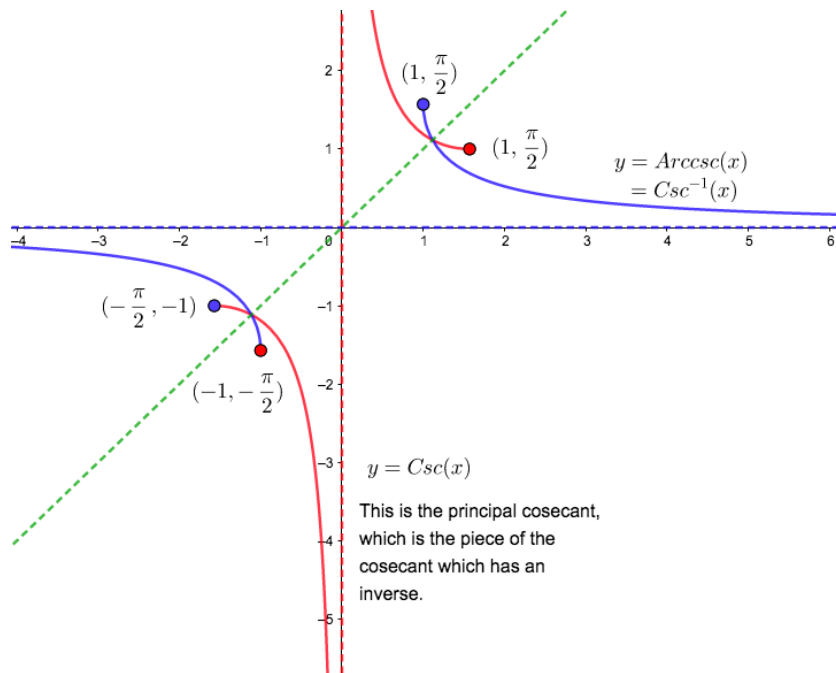


Figure 11: The Cosecant Function and its Inverse