

Conditions and Test Statistics for Statistical Tests

Inference About: (Number of Samples)	Name of Test	Test Statistic	Conditions	Nonparametric Option
Proportion (1 sample)	Z-Test	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	<ul style="list-style-type: none"> • Random sample • n is large (at least 10 successes and 10 failures) 	None
Proportion (2 sample)	2-Sample Z test	$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}}$ <p>Note: $\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$</p>	<ul style="list-style-type: none"> • Two independent random samples • n_1 and n_2 large (at least 10 successes and 10 failures) 	None
Proportion (1 sample)	Binomial Test	$\Pr(X) = \binom{n}{X} p^X (1-p)^{n-X}$	<ul style="list-style-type: none"> • Every outcome is either a success or failure • Number of trials is fixed • Separate trials are independent • Probability of success is the same for each trial 	None
Test to determine if distribution of a nominal variable matches predicted distribution (1 sample)	χ^2 Goodness of Fit Test	$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ <p>$df = \text{num. of cat.} - 1$</p>	<ul style="list-style-type: none"> • Random sample • All expected counts at least 5 	None

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Relationship Between 2 Categorical Variables	χ^2 Contingency Test	$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$ $df = (r - 1)(c - 1)$	<ul style="list-style-type: none"> • Random sample(s) • No more than 20% of cells have counts less than 5, and all are at least 1 	Fisher's Exact Test
Relationship Between 2 Categorical Variables: Special Case 2X2 table	Fisher's Exact Test	$P = \sum \frac{(R_1!R_2!)(C_1!C_2!)}{N!a_{1,1}a_{1,2}a_{2,1}a_{2,2}}$ <p>$N =$ sum of all entries. The sum is taken over all tables with the same row and column total with P values at least as extreme as the first</p>	<ul style="list-style-type: none"> • Random sample(s) • Used when expected count levels are not met for the χ^2 test 	
Mean (1 sample, unpaired)	One-sample t-Test	$t = \frac{(\bar{x} - \mu_0)}{\frac{s}{\sqrt{n}}}$ $df = n - 1$	<ul style="list-style-type: none"> • Random sample • Population has normal distribution OR sample size is large ($n > 30$) 	Sign Test
Median (1 sample) Nonparametric Test	Sign Test	$S = \text{smaller} \begin{cases} \text{number of + signs} \\ \text{number of - signs} \end{cases}$ $p\text{-value} = \sum_{i=0}^S (0.5)^i (0.5)^{(N-i)}$ <p>where $N =$ total number of experimental units</p>	<ul style="list-style-type: none"> • Random samples • Independent observations • Variable needs to be ordinal 	

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Mean (1 or 2 sample Matched Pairs)	One-Sample t-Test	$t = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n_d}}}$ $df = n_d - 1$	<ul style="list-style-type: none"> Paired data from a random sample Population of differences normally distributed OR number of pairs is large ($n_d > 30$) 	Wilcoxon Signed Rank Test
Median (1 sample) Nonparametric Test	Wilcoxon Signed Rank Test	W = smaller { <ul style="list-style-type: none"> sum of positive ranks sum of negative ranks 	<ul style="list-style-type: none"> Paired sample Independent observations Response variable must be at least ordinal Distribution of differences symmetric about median 	
Mean (2 samples) (Variances the same)	Two-Sample (pooled) t-Test	$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$ $s_p = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1 + n_2 - 1}}}$ $df = n_1 + n_2 - 2$	<ul style="list-style-type: none"> Random, independent samples from two populations Both population distributions normally distributed OR both samples large ($n_1, n_2 > 30$) Variances need to be the same* <p>*Check the results of Levene's test to see if this condition is violated. If yes, use the adjusted results</p>	Mann Whitney U-Test
Means (2 samples) (Variances NOT the same)	Welch's Test	$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	<ul style="list-style-type: none"> Same as above EXCEPT variances can be unequal 	Mann Whitney U Test

Inferences About: (Number of Samples)	Name of Test	Test Statistic	Conditions	Nonparametric Options
<p>Equality of Two Population Distributions</p> <p>Nonparametric Test</p> <p>(Can be used to compare means or medians only if two distributions have the same shape)</p>	Mann Whitney U test	<p>U= smaller (U_1, U_2) where</p> $U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1$ $U_2 = n_1n_2 + \frac{n_2(n_2+1)}{2} - R_2$	<ul style="list-style-type: none"> • Random samples from two populations • Response (dependent) variable must be at least ordinal • Independent samples 	
Means (Multiple Independent Samples)	ANOVA	$F = \frac{MS_{groups}}{MS_{error}}$ $= \frac{\frac{SS_{groups}}{k-1}}{\frac{SS_{error}}{n-k}}$ <p>n=number of subjects k= number of groups OR</p> $F = \frac{MS_{between}}{MS_{within}}$	<ul style="list-style-type: none"> • k random samples • Each population is normally distributed • Variances the same in all populations (robust if this is condition is violated) 	Kruskal Wallis Test
<p>Medians (Multiple Independent Samples)</p> <p>Nonparametric Test</p>	Kruskal Wallis Test	$H = \frac{12}{N(N+1)} \sum_{j=1}^k \frac{R_j^2}{n_j} - 3(N+1)$	<ul style="list-style-type: none"> • k random samples • Independent samples • Response (dependent) variable must be at least ordinal • Distributions have same shape in each population 	

Inference About: (Number of Samples)	Name of Test	Test Statistic	Conditions	Nonparametric Options
Means (Multiple Dependent Samples)	Repeated Measures ANOVA	$F = \frac{MS_{conditions}}{MS_{error}}$ OR $F = \frac{MS_{between}}{MS_{residual}}$	<ul style="list-style-type: none"> • k random samples • Same variable measured under different conditions on same subjects • Test variable follows a multivariate normal distribution • *Variances of all difference scores are equal <p>Look at results of Mauchly's test. If significant, use Greenhouse Geisser correction if $GG < .75$, otherwise use Huynh-Feldt correction</p>	Friedman's Test
Means (Multiple time points of dependent samples) Nonparametric Test	Friedman's Test	$Q = \frac{12n}{k(k+1)} \cdot \sum_{j=1}^k \left(\bar{r}_{ij} - \frac{k+1}{2} \right)^2$	<ul style="list-style-type: none"> • k random samples • Same variable measured under different conditions on same subjects • Variances of difference scores are equal 	
Means (Examining the effects of two explanatory (independent) variables)	Two Way ANOVA	Calculate F_A, F_B, F_{AB} where $F_A = \frac{MS_A}{MS_{within\ cell}}$	<ul style="list-style-type: none"> • k random samples • Each population is normally distributed • Variances the same in all populations (robust if not) 	

Inference About: (Number of Samples)	Name of Test	Test Statistic	Conditions	Nonparametric Options
Slope (Testing to see if slope of linear relationship is different from a given parameter value)	Linear Regression t-Test	$t = \frac{b-\beta}{SE_b}$ where SE_b is the standard error of the slope $SE_b = \sqrt{\frac{\sum(y_i - \hat{y})^2}{n-2}}{\sum(x_i - \bar{x})^2}$ $df = n - 2$	<ul style="list-style-type: none"> • Random sample • (x, y) pairs independent • There is a linear relationship between x and y • For each x, the y values are normally distributed with equal variances • For each y, the x values are normally distributed with equal variances 	Spearman's Rank Correlation
Correlation (Test to see if the correlation between two numerical values is significant)	Significance test on Pearson's Correlation Coefficient	$t^* = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ $r = \text{Pearson's correlation coefficient}$ $r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$	<ul style="list-style-type: none"> • Random sample • (x, y) pairs independent • There is a linear relationship between x and y • For each x, the y values are normally distributed with equal variances • For each y, the x values are normally distributed with equal variances 	Spearman's Rank Correlation
Correlation (Test to determine if the correlation between two variables (must be at least ordinal) is significant) Nonparametric Test	Significance Test for Spearman's Rank Correlation	$t = \frac{r_s\sqrt{n-2}}{\sqrt{1-r_s^2}}$ r_s is computed in the same way as r , but applied to ranks	<ul style="list-style-type: none"> • Random sample • (x, y) pairs are independent • Relationship between the variables is monotonic 	

Post-hoc Test	Test Statistic	Conditions
Tukey's HSD (For all possible comparisons)	$HSD = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{\frac{MS_w}{n}}}$ $MS_w = \text{Mean Squares Within}$ $n = \text{number per treatment group}$	<ul style="list-style-type: none"> • Met Assumptions of ANOVA
Fisher's LSD (Does not correct for multiple comparisons)	$LSD_{ij} = t_{\alpha, dfW} \cdot \sqrt{MS_w \cdot \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$	<ul style="list-style-type: none"> • Met assumptions of ANOVA
Dunn's Procedure / Bonferoni Correction	$t = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{MS_w \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$ OR $F = \frac{(\bar{x}_i - \bar{x}_j)^2}{MS_w \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$ Use $\alpha_i = \frac{\alpha}{C}$ C= Number of Comparisons	<ul style="list-style-type: none"> • Met assumptions of ANOVA
Cohen's D Effect Size Calculator	$d = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{S_1^2 + S_2^2}{2}}}$ if equal sample sizes Otherwise $d = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}}}$	<ul style="list-style-type: none"> • Met assumptions of 2 sample t-test
Simple Effects Test	$t = \frac{\overline{AB}_{1j} - \overline{AB}_{2j}}{\sqrt{MS_{err} \cdot \left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$	<ul style="list-style-type: none"> • Met assumptions of 2-way ANOVA