Simpsons Rule! An Examination of Math in *The Simpsons*

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An Examination of Math in *The Simpsons*

An Honors Thesis
Presented to the Department of Mathematics
Bates College
in partial fulfillment of the requirements for the
Degree of Bachelor of Arts
by

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Lewiston, Maine

December 16, 2006

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Acknowledgments

I would like to begin by thanking my thesis advisor, Professor Johnathan I. Q. Frink, Jr.

My major advisor, Professor Thomas Simpson, ruled (well, at least approximately) my college years and helped me integrate mathematics into my life.

My roommate E.H. Simpson was able to correct a higher percentage of the errors he found in the first draft of this thesis than the percentage of the errors I found that I was able to correct in the first draft. The same is true about the second draft. Yet, paradoxically, when considering the two drafts overall, my percentage was higher than his.

My friends, Jessica and Ashlee, were of great assistance to me in figuring out some of the more complicated mathematics in this paper.

Finally, I would like to thank all the little people, including Bart, Lisa, and Maggie.

Introduction

This is a senior thesis so excellent, so famous, so sublime that it truly needs no introduction.[1]

[1] And yet you are currently reading the Introduction. What a paradox.

CHAPTER 1

Pi, or "Mmm... pie."

In an episode that first aired on March 11, 2001, Professor Frink finds himself unable to quiet down a rowdy audience of scientists. To get their attention, he finally shouts out, "Pi is exactly three!"

We now proceed to disprove this outrageous statement.

DEFINITION. A real number r is *transcendental* if there does not exist any polynomial $P(x)$ with integer coefficients such that $P(r) = 0$.

THEOREM 1.1. *The number π is transcendental.*[1]

PROOF. The proof of this theorem is just slightly beyond the scope of this thesis. It was first accomplished by Ferdinand von Lindemann in 1882. Interested readers should see his paper in [**2**].

□

THEOREM 1.2. *The number 3 is not transcendental.*

PROOF. According to our definition above, it is sufficent to find a polynomial $P(x)$ with all integer coefficients such that $P(3) = 0$.

Consider $P(x) = x^{132} - 3x^{131} - 2x^{74} + 6x^{73} - x^2 + 9$. Clearly, $P(x)$ is a polynomial and its coefficients are all integers. And even a simpleton can see at a glance that $P(3) = 0$.

□

COROLLARY 1.3. *The number π is not exactly 3.*

PROOF. We saw in Theorem 1.1 that π is transcendental and in Theorem 1.2 that 3 is not transcendental. A number can not be both transcendental and not transcendental[2], so π must not be equal to 3.

□

[1] Notice how we avoided the terrible sin of beginning a sentence with a number or symbol.

[2] This assumes that you believe in the Law of the Excluded Middle. If not, you may be in for a difficult time ahead.

We shouldn't think that all the π-related math on *The Simpsons* is bogus. For example, in a "Treehouse of Horror" episode from October 30, 1995, we see the following equation, which, as we will learn shortly, is true.

$$e^{i\pi} = -1 \tag{1.1}$$

THEOREM 1.4. *For any $\theta \in \mathbb{R}$, we have $e^{i\theta} = \cos\theta + i\sin\theta$.*

PROOF. Oh for crying out loud, this is so darned obvious I won't waste my time, but if you must see a proof, please read [**1**] or any other good text on complex analysis.

□

COROLLARY 1.5. *The equation $e^{i\pi} = -1$ is true.*

PROOF. If we let $\theta = \pi$ in Theorem 1.4, we obtain the following.

$$e^{i\pi} = \cos\pi + i\sin\pi = -1 + i\cdot 0 = -1$$

□

REMARK. We note that Equation (1.1) would look so much more beautiful if 1 were added to both sides to give the following.

$$\boxed{e^{i\pi} + 1 = 0}$$

This single equation captures what many consider to be the five most important numbers in mathematics. Good times.

CHAPTER 2

FLT - Fried Lettuce and Tomato Sandwich?

In two episodes (which first aired on October 30, 1995 and September 20, 1998), we see a statement of the form $a^{12} + b^{12} = c^{12}$, where a, b, and c are natural numbers. These equations are the following.

$$1782^{12} + 1841^{12} = 1922^{12} \tag{2.1}$$

$$3987^{12} + 4365^{12} = 4472^{12} \tag{2.2}$$

This may call to mind a little-known theorem.

THEOREM 2.1 (Fermat's Last Theorem, or FLT). *The equation $x^n + y^n = z^n$ has no non-zero integer solutions for $n > 2$.*

PROOF. I have a marvellous proof of this, but the grant funding to produce this manual is too small for me to present it here. However, interested readers may wish to see [**3**].

□

A consequence of this theorem, of course, is that equations (2.1) and (2.2) must be incorrect. But how might one determine this without recourse to FLT? Well, if one had lots of spare time late at night (as you very well may, dear reader), one could just go ahead and multiply them out. This author, on the other hand, has a very active social life. So, we instead recall the following.

LEMMA 2.2. *The product of any two odd integers is an odd integer. The product of any two even integers is an even integer.*

PROOF. Look in your notes or text from Math Camp. This is one of many similar results we proved back in those good old days.

□

COROLLARY 2.3. *An odd integer raised to any positive integer power is an odd integer. That is, for odd $z \in \mathbb{Z}$, z^m is odd for any $m \in \mathbb{N}$.*

PROOF. By induction. Suppose z is any odd integer.
Base case: Since z is odd, we know that $z^1 = z$ is odd.

Inductive step: Suppose z^n is odd for some $n \in \mathbb{N}$ (our inductive hypothesis). We must show that z^{n+1} is also odd.

Well,... $z^{n+1}=z^n \cdot z$ by basic properties of exponents.

By our inductive hypothesis we know that z^n is odd, and we began by assuming that z is odd. So, $z^n \cdot z$ is the product of two odd integers and is therefore odd by Lemma (2.2).

□

COROLLARY 2.4. *An even integer raised to any positive integer power is an even integer.*

PROOF. This proof is nearly identical to the one above. To reduce the printing cost of this thesis and conserve some of the planet's natural resources, we omit it.

□

We now look back to equation (2.1), $1782^{12} + 1841^{12} = 1922^{12}$, and see that we have an two even integers, 1782 and 1922, raised to positive integer powers; thus, by Corollary 2.4, each of the results is even. Similarly, by Corollary 2.3, 1841^{12} is odd.

Now all that remains is to invoke one of the most powerful theorems known to mankind.

THEOREM 2.5. *The sum of an even integer and an odd integer is an odd integer.*

PROOF. The proof of this theorem is so far beyond the scope of this thesis as to be nearly incomprehensible to a person of your intellect. Some of the greatest math students in the history of Colby College are currently devoting all their talents and energies to a proof, even to the extent that they have had to reduce their daily hours spent playing Dungeons and Dragons from sixteen to just two. Fortunately, they are unemployed and living in their mothers' basements, so they have little else to do.

□

By applying Theorem 2.5 to the left-hand side of Equation (2.1), we see that this left-hand side must be odd. But we already found above that the right-hand side is even. Thus, this equation cannot be true.

Alas, this same argument cannot be applied to Equation (2.2). [Why not?] However, by using a different base for our modular arithmetic, we can arrive at the same conclusion. This, my loyal and persevering reader, is left on your shoulders. So put away your video games and your crazy rock and roll records and get to work, you slacker.

Appendix A. For Very Interested Readers Only

I had my appendix removed when I was a little boy, so I cannot include it here. You are free to visit my office to see it, though; it's in a jar on the shelf.

Bibliography

[1] Lars Ahlfors, *Complex Analysis*, McGraw-Hill, New York, 1979.
[2] Ferdinand von Lindemann, Über die Zahl π, *Mathematische Annalen* **20** (1882), 213–225.
[3] Andrew Wiles, Modular Elliptic Curves and Fermat's Last Theorem, *Ann. Math.* **141** (1995), 443-551.