## Lesson Three: Proclamations and Proofs

**Definition.** An integer $m$ is *even* if $m = 2j$ where $j$ is an integer.

**Theorem 1.** *The sum of any two even integers is an even integer.*

*Proof.* Suppose $m$ and $n$ are even integers.
By the definition of even, $m = 2j$ and $n = 2k$ where $j$ and $k$ are integers.
Therefore, $m + n = 2j + 2k = 2(j + k)$.
Since the integers are closed under addition, $j + k$ is also an integer.
So, $m+n$ is twice another integer $(j+k)$, meaning $m+n$ is even, as desired. □

**Theorem 2.** *The equation $x^n + y^n = z^n$ has no non-zero integer solutions for $n > 2$.*

*Proof.* I have a marvellous proof of this, but the page is too small to contain it. □

**Theorem 3.** *The number 8675309 is prime.*

*Proof.* Just ask Jenny. □

**Definition.** A *mathematician* is a device for turning coffee into theorems. *[attributed to Paul Erdős]*

*Note.* Begin laughing now.

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## Exercise Three: Proclamations and Proofs

**Theorem 1.** *The product of any two even integers is an even integer.*

*Proof.* `[Try this on your own before looking at the solutions.]` □

**Definition.** An integer $k$ is *odd* if $k = 2j + 1$ where $j$ is an integer.

**Theorem 2.** *The sum of any two odd integers is an even integer.*

*Proof.* `[Try this on your own before looking at the solutions.]` □

**Theorem 3.** *Every even integer greater than two is the sum of two primes.*

*Proof.* `[Let us know right away if you find a proof to this one!]` □