Lesson Four: Set Notation, Greek and Hebrew Letters

A fringe benefit of being a math major is that you will soon learn much of the Greek alphabet and even some Hebrew. Plus, math is a language unto itself.

Let $S = \{\pi, \delta, \Delta\}$ and $T = \{\Omega\}$.
Then $S \cup T = \{\pi, \delta, \Delta, \Omega\}$, $S \cap T = \emptyset$, and $S \times T = \{(\pi, \Omega), (\delta, \Omega), (\Delta, \Omega)\}$.

In set notation the interval $(1, 3]$ is written $\{x \in \mathbb{R} : 1 < x \leq 3\}$.

Is infinity a number? No! It's more like a state of mind ... But $\mathbb{R} = (-\infty, \infty)$.

Notation. We write $A \subseteq B$ if and only if for each $x \in A$, we also have $x \in B$.

Notation. We write $A \not\subseteq B$ if there exists $x \in A$ such that $x \notin B$.

So, $\mathbb{N} \subseteq \mathbb{R}$ but $\mathbb{R} \not\subseteq \mathbb{N}$.

Power sets are lots of fun.
If $X = \{7, \{e\}\}$, then $\mathcal{P}(X) = \{\emptyset, \{7\}, \{\{e\}\}, \{7, \{e\}\}\}$.

Notation. The cardinality of $\mathbb{N}$ is referred to as $\aleph_0$, which is read "aleph naught."

Exercise Four: Set Notation, Greek and Hebrew Letters

If $A = \{\heartsuit, \spadesuit\}$ and $B = \{\epsilon, \theta\}$, then
$A \cup B =$ [`Fill in the blank then check the solutions.`]
$A \cap B =$ [`Fill in the blank then check the solutions.`]
$\mathcal{P}(A \cap B) =$ [`Fill in the blank then check the solutions.`]

Theorem 1. *If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.*

Proof. We assume $A \subseteq B$ and $B \subseteq C$, and we want to show $A \subseteq C$.
Suppose $x \in A$. We must show that $x \in C$ also.
Since $A \subseteq B$, we know $x \in B$.
Similarly, since $B \subseteq C$, we also know $x \in C$.
But this means $A \subseteq C$, as desired. □

Theorem 2. *If $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.*

Proof. `Try this one on your own before looking at the solutions.`
□

Puzzling Power Sets

[`The sets below should have one, two, and four elements, respectively.`]
$\mathcal{P}(\emptyset) =$
$\mathcal{P}(\mathcal{P}(\emptyset)) =$
$\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) =$