Lesson Five: Alignment, Part Two - Labels and Annotation

Suppose that $f(x) = 1/x$. Compute $f'(x)$ using the limit definition of the derivative.

(1)	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$	This is the limit definition.
(2)	$= \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$	
(3)	$= \lim_{h\to 0} \frac{\frac{x}{(x+h)(x)}-\frac{x+h}{(x+h)(x)}}{h}$	Get a common denominator.
(4)	$= \lim_{h\to 0} \frac{\frac{x-(x+h)}{(x+h)(x)}}{h}$	Combine the fractions.
(5)	$= \lim_{h\to 0} \frac{\frac{-h}{(x+h)(x)}}{h}$	Cancel the x's.
(6)	$= \lim_{h\to 0} \frac{\frac{-1}{(x+h)(x)}}{1}$	Cancel the h's.
(7)	$= \frac{-1}{(x)(x)}$	
(8)	$= -\frac{1}{x^2}$	

Note that in (1), some professors will (properly!) like you to include the phrase "provided this limit exists."

The key algebra step was to get a common denominator in (3).

A common error is to forget the parentheses around the $x+h$ in (4).

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Claim. *The real number 2 is equal to the real number 1.*

Proof.

(1)	$a = b$	Suppose $a, b \in \mathbb{R}$ are non-zero and that $a = b$.
(2)	$a^2 = ab$	What's the harm in multiplying each side by a?
(3)	$a^2 - b^2 = ab - b^2$	And it's fair to subtract b^2 from each side, no?
(4)	$(a-b)(a+b) = b(a-b)$	Just factoring. Nothing fishy here.
(5)	$a + b = b$	Cancel the factor $(a-b)$ from each side.
(6)	$b + b = b$	Recall from (1) that $a = b$, so we replace a by b.
(7)	$2b = b$	Who could possibly object to this?
(8)	$2 = 1$	In (1) we defined $b \neq 0$, so we can divide by b.

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