Lesson Eight: Over/Under and Boxes

Overline and Underline

In some cases, we like to underline important notions, such as $\underline{\text{cardinality}}$.

A common notation for the cardinality of the set A is $\overline{\overline{A}}$.

Overbrace and Underbrace

Can't remember those pesky exponent rules? Happens to all of us. [1]

Suppose $m, n \in \mathbb{N}$. Then we have the following

$$a^m a^n = (\underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}})(\underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}) = (\underbrace{a \cdot a \cdot \ldots \cdot a}_{m+n \text{ times}}) = a^{m+n}$$

We all know that $e^w = 1 + w + \frac{w^2}{2!} + \frac{w^3}{3!} + \ldots$ for all $w \in \mathbb{R}$. But what happens if we're so reckless as to let $w = ix$, where $i = \sqrt{-1} \in \mathbb{C}$? Let's see.

$$\begin{aligned} e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \ldots \\ &= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} + \ldots \\ &= \underbrace{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots}_{\text{the series for } \cos x} + i\underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right)}_{\text{the series for } \sin x} \\ &= \cos x + i \sin x \end{aligned}$$

Boxes

A consequence of the above formula is the following, which relates five of the most important numbers in mathematics and thus deserves its own box.

$$\boxed{e^{i\pi} + 1 = 0}$$

Exercise Eight: Over/Under and Boxes

Some colleges to which one might apply are the following.

$\underbrace{\text{Yale, Colby}}_{\text{safety schools}}$, Bowdoin, Wisconsin, $\underbrace{\text{Harvard, Bates}}_{\text{top choices}}$

More on those easily forgotten exponent rules. As before, we let $m, n \in \mathbb{N}$.

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ times}}}{\underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}} = a^{m-n}$$

$$(a^m)^n = \underbrace{(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}})(\underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}}) \ldots (\underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}})}_{n \text{ times}} = a^{mn}$$

If A and B are finite sets, then $A \cap B = \emptyset \iff \overline{\overline{A}} + \overline{\overline{B}} = \overline{\overline{A \cup B}}$

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[1] Well, not to me, of course.