Note on the analogy between inertial and electromagnetic forces

Mark D. Semon
Department of Physics and Astronomy, Bates College, Lewiston, Maine 04240

Glenn M. Schmieg
Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201

(Received 19 February 1980; accepted 16 September 1980)

If we let ω be the instantaneous angular velocity of a frame rotating about a point fixed in a Newtonian frame,[1] then it is well known that an observer in the rotating frame will observe three inertial forces:

$$\mathbf{F}_1 = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}), \quad (1)$$

$$\mathbf{F}_2 = 2m(\mathbf{v} \times \boldsymbol{\omega}), \quad (2)$$

and

$$\mathbf{F}_3 = -m\dot{\boldsymbol{\omega}} \times \mathbf{r}. \quad (3)$$

It has been noted previously[2] that one way to visualize the effects of the centrifugal force $\mathbf{F}_1$ and the Coriolis force $\mathbf{F}_2$ is to study the effect of certain electric and magnetic forces on test charges in a Newtonian frame. In other words, there exist electric and magnetic forces in Newtonian frames whose effect on charged particles exactly mimics the effects of centrifugal and Coriolis forces in rotating frames of reference. Using this analogy makes the effects of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ easier to understand, since most students are familiar with basic electric and magnetic phenomena by the time they study noninertial reference frames.

The question naturally arises: is there an electromagnetic force that mimics the effect of the transverse force $\mathbf{F}_3$? The purpose of this note is to supplement Coisson's paper[2] by pointing out that the force $\mathbf{F}_3$ is completely analogous to the force associated with an induced emf. To this end, we begin with the electromagnetic case, and then develop this and the inertial case concurrently.

Consider a Newtonian frame with positive test particles of charge q. By measuring the acceleration of these particles under various circumstances, and using the Lorentz force equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (4)$$

we can assign an electric and magnetic field vector to every point, thus defining an electromagnetic field.

A corresponding procedure can be used in a frame of reference rotating with a constant angular speed ω about the z axis of some Newtonian frame. In this case there are no external fields, and by measuring the acceleration of test masses under various circumstances and using the force Eqs. (1) and (2), a rotating observer can assign an $\boldsymbol{\omega}$ vector to every point, thus defining an $\boldsymbol{\omega}$ field. In the present case this field is found to be uniform, and to point along the z axis. We note in passing that this "inertial field" is just as real in the rotating frame as the electromagnetic field is in the Newtonian frame, since both are defined in the same way.

The effect of the inertial field in the rotating frame is imitated in the Newtonian frame if the applied electric and magnetic fields are

$$\mathbf{E} = (m/q)\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad (5)$$

and

$$\mathbf{B} = -(2m/q)\boldsymbol{\omega}. \quad (6)$$

A positive charge, acted upon by these fields in a Newtonian frame, will behave exactly like a particle of mass m in a frame rotating with a constant angular velocity $\boldsymbol{\omega}$.[3] In this way the effects of the centrifugal and Coriolis forces can be easily visualized.

Next suppose that the magnetic field of Eq. (6) starts to vary with time. For simplicity assume it to be increasing in the positive z direction. In this case an electric field $\boldsymbol{\epsilon}$ is induced that is tangent to any circle in the xy plane with its center at the origin. This induced field is nonconservative, and supplies the only contribution to the work done on a charged particle carried around a closed path in the xy plane containing the origin. This occurs because the electric field of Eq. (5) is conservative ($\nabla \times \mathbf{E} = 0$), and the force associated with the magnetic field of Eq. (6) is always perpendicular to the path. Thus the work done on a charged particle carried around the path described above is

$$W = q \oint \boldsymbol{\epsilon} \cdot d\mathbf{l}. \quad (7)$$

The integral in Eq. (7) is called the induced emf, and can be evaluated using Stokes's theorem and the Maxwell equation

$$\nabla \times \boldsymbol{\epsilon} = -\frac{\partial \mathbf{B}}{\partial t} \quad (8)$$

to give

$$W = -q \int \dot{\mathbf{B}} \cdot \hat{n}\, d\sigma \quad (9)$$

$$= -q\dot{B}A, \quad (10)$$

where $\dot{\mathbf{B}}$ is the time derivative of the magnetic field described above. The integral in Eq. (9) is over the area A enclosed by the path in the xy plane around which the charge q is carried.

The corresponding situation in the rotating frame is achieved if we let $\boldsymbol{\omega}$ increase in the positive z direction. This means, of course, that the rotating frame is increasing its

 0002-9505/81/070689-02$00.50

angular speed ω about the z axis, as seen by an inertial observer. A rotating observer sees a uniform $\boldsymbol{\omega}$ field parallel to the positive z axis that is increasing with time, and thus also measures the transverse force of Eq. (3) in addition to the other inertial forces of Eqs. (1) and (2). The transverse force is tangent to any circle in the xy plane centered at the origin. The work done on a test mass carried around a closed path in the xy plane containing the origin is

$$W = \oint (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3) \cdot \mathbf{dl}. \tag{11}$$

The integral of $\mathbf{F}_1$ around the closed path is zero since this force is conservative ($\nabla \times \mathbf{F}_1 = 0$). The Coriolis force $\mathbf{F}_2$ is always perpendicular to the path, and so does not contribute to the integral. Thus we only need evaluate the contribution of $\mathbf{F}_3$ in Eq. (11). Using the identity

$$\dot{\boldsymbol{\omega}} \times \mathbf{r} \cdot \mathbf{dl} = \dot{\boldsymbol{\omega}} \cdot \mathbf{r} \times \mathbf{dl}, \tag{12}$$

the integral in Eq. (11) becomes

$$W = -m \oint \dot{\boldsymbol{\omega}} \cdot (\mathbf{r} \times \mathbf{dl}). \tag{13}$$

Since $\dot{\boldsymbol{\omega}}$ is uniform and is parallel to $\mathbf{r} \times \mathbf{dl}$ this becomes

$$W = -m\dot{\omega} \oint |\mathbf{r} \times \mathbf{dl}|. \tag{14}$$

The integral in Eq. (14) is recognized as twice the area enclosed by the path. Thus

$$W = -2m\dot{\omega}A. \tag{15}$$

Equation (15) is exactly what we would have predicted from the electromagnetic analogy, since by Eqs. (6) and (10),

$$\begin{aligned} W &= -q\dot{B}A \\ &= -q(2m/q)\dot{\omega}A \\ &= -2m\dot{\omega}A. \end{aligned}$$

Thus we see that the transverse force $\mathbf{F}_3$ in a rotating frame behaves exactly like the force associated with an induced emf in a Newtonian frame. Both forces have the same direction, are nonconservative, and result in analogous expressions for the work. Because of this, the force associated with an induced emf in a Newtonian frame gives a very clear picture of the transverse force in a rotating frame of reference.

[1] By "Newtonian frame" we mean an inertial frame of reference, in which Newton's laws assume their simplest form.

[2] R. Coisson, Am. J. Phys. **41**, 585 (1973).

[3] This observation forms the basis of Larmor's theorem.

Fresnel–Arago laws for interference in polarized light: A demonstration experiment

M. Henry
Laboratoire d'Optique, Université Pierre et Marie Curie, 4 place Jusseiu, 75230 Paris, Cedex 05, France
(Received 23 July 1979; accepted 18 February 1980)

About one century ago, Fresnel and Arago investigated the influence of polarization of light on interference patterns.[1] Most textbooks include a discussion of these laws,[2-4] but few demonstration experiments can be found in the literature.[5-8]

Some years ago, Fortin[7] described an experiment making use of laser light and of a two-slit interferometer. We refer the reader to this note for a discussion of the problems involved in actual realization of a setup. As he points out the main problem is to simultaneously have a large distance between the slits, for convenient setting of polarizers, and a fringe spacing large enough for convenient observation—two conflicting requirements.

We meet, at least partially, these two requirements by using a grating as a beam splitter, and by observing the interference fringes in its conjugate plane. The grating generates a number of mutually coherent beams sufficiently separated from each other, and we select two of these beams to carry out the experiment. In this method of beam splitting, the amount of light available in the fringe plane is much more than in the wave-front division method used by Fortin,[7] and this permits a high magnification for convenient observation at a reasonable distance.

The unexpanded laser beam (Fig. 1) impinges on a grating located in the front focal plane of lens L_1. In the back focal plane of L_1, we observe the grating diffraction spectrum. A screen S with two apertures selects two diffracted orders. Punched computer cards constitute convenient—and cheap—screens. Any amplitude or phase grating will serve the purpose: one of ours was a Ronchi ruling of 25 lines/mm.

The back focal plane of L_1—and screen plane—coincide with the front focal plane of L_2. The interference fringes produced by the two beams are situated in the back focal plane of L_2, which is also the image plane of the grating. These fringes are magnified, and projected onto an observation screen E by a microscope objective O. We list below some numerical values, but they are not critical:

laser: 2-mW cw He–Ne ($\lambda = 632.8$ nm);

grating: Ronchi ruling with 25 l/mm;

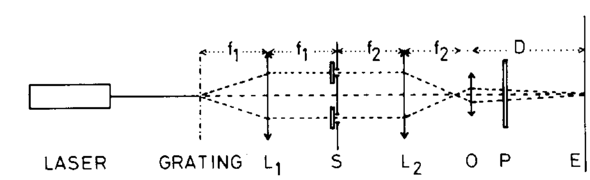

Fig. 1. Experimental setup. L_1, L_2: lenses; O: microscope objective; S: screen with two holes; E: observation screen; P: polarizer.

 0002-9505/81/070690-02$00.50