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D0, in the Tevatron. The data agree spectacularly well with the theoretical prediction of QCD.

QCD and Hadron Masses

Quantum chromodynamics is part of the standard model of elementary particles. It occupies a special place in the standard model because it gives a thoroughly consistent mathematical theory of the natural world. Given the quark masses and the strong coupling (at a single, fixed distance), QCD should explain strong interaction phenomena at all distance scales, all energies, all temperatures of the universe, and so on. This is exceptional and awe inspiring; QCD calls out to scientists for tests of the most exacting standards.

At high energies or, equivalently, at short distances, asymptotic freedom makes theoretical analysis of QCD tractable and reliable. As Figs. 2 and 3 attest, the confrontation of theory and experiment is splendid. At longer distances, however, the confining force plays a strong role, and theoretical physicists must turn to other methods to extract information from QCD.

For example, from QCD one should be, and in principle is, able to compute the masses of all hadrons. But the most relevant distance for the hadron masses is the hydronic radius, $r \approx 0.5$ fm, where the strong coupling $\alpha_s(r)$ is large. At these distances one must treat the chromoelectric and chromomagnetic fields fully, instead of one gluon at a time. Moreover, in a quantum field theory, like QCD, many field configurations are important. The high strength of the coupling and the need for many field configurations presents a tremendous challenge. The most promising line of attack, suggested by Kenneth Wilson in 1974, employs large-scale computer calculations. The computer is programmed to generate (numerical representations of) the most important configurations. It is then not too difficult to obtain the masses from various averages over the set of configurations. The difficulty lies instead in the generation of the fields. Physicists have been able to complete the computer calculations only with certain undesirable, but computationally frugal, approximations.

To succeed in calculating the hadron masses from QCD will be a grand achievement, for it will prove that quantum chromodynamics explains the strong interactions from the tiniest distances probed by particle accelerators out to the diameter of the atomic nucleus.

See also: Baryon Number; Boson, Gauge; Color Charge; Gauge Theories; Hadron; Quantum Electrodynamics; Quark; Quark Confinement

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Andreas S. Kronfeld

QUANTUM ELECTRODYNAMICS

Quantum electrodynamics (QED) is the theory that describes light (electromagnetic radiation) and its interaction with matter (i.e., with electrons and other charged particles). It was formulated in the late 1920s by P. A. M. Dirac, Werner Heisenberg, Pascual Jordan, and Wolfgang Pauli, and completed in the early 1950s by Freeman J. Dyson, Richard P. Feynman, Julian Schwinger, and Sin-itiro Tomonaga. Although they worked independently, both Schwinger and Tomonaga constructed the theory in terms of relativistic quantum fields, while Feynman used a diagrammatic approach based on trajectories of particles in spacetime (now called Feynman diagrams). Feynman's method proved to be an incredibly effective computational tool, which Dyson then showed was derivable as a power series expansion known as perturbation theory from the formulations of Schwinger and Tomonaga. Dyson also proved that various divergences (i.e., infinite values for calculated quantities), present in both the field-theoretic and diagrammatic approaches, could be eliminated (to all orders in perturbation theory) by redefining how the mass and charge parameters appearing in the theory were related to the values observed in the lab. This method of handling the divergences (now called renormalization) had been explored earlier by Dirac, Heisenberg, Pauli, Victor Weisskopf, Hendrik A. Kramers, and Sidney Michael Dancoff and has continued to be improved on to the present.

QED developed as the result of an impressive interplay between theory and experiment. It was, in part, the newly developed technology of microwaves that made possible the extremely precise measurements of the spectrum of hydrogen (the Lamb Shift) by Willis E. Lamb and Robert C. Retherford, and the magnetic moment of the electron by Polykarp Kusch and Henry M. Foley. Both of these results, published in 1947, stimulated rapid theoretical advances that in turn stimulated experimenters to develop new techniques for making even more precise measurements. At present, although there is still room for improvement, both theory and experiment agree to a remarkable degree of accuracy and over an incredible range of energies. For example, the agreement between experiment and theory has been confirmed to within parts per million (ppm) in measurements on atomic spectra, which detect energies of order 10^{-6} eV, and to within 1 percent in electron-positron colliding beam experiments, which occur at energies of around 100 GeV. Thus, theory and experiment are in excellent agreement over an energy range of about fifteen orders of magnitude. At energies higher than about 100 GeV, the weak and strong interactions become important and QED must be viewed as part of a more comprehensive theory that includes them and perhaps gravity as well.

Background

At the beginning of the nineteenth century there were two different theories of light: a particle theory and a wave theory. The particle theory fell into disfavor after interference effects were demonstrated in the early 1800s. It was almost totally set aside in the late 1800s when James Clerk Maxwell showed that all electric, magnetic, and optical phenomena could be derived from four equations (Maxwell's equations), and that these equations *predicted* electromagnetic waves that traveled with speed $c = 2.9979 \times 10^8$ m/s. Since this value was close to the speed of light that had been measured earlier, Maxwell hypothesized that light itself was an electromagnetic wave and, by the early 1900s, the electromagnetic wave theory of light was well established. In fact, the visible, infrared, ultraviolet, radio, x ray, gamma ray, and so on all can be viewed as electromagnetic waves of different wavelengths; in what follows, we use the word "light" to refer to the whole electromagnetic spectrum rather than to just that part visible to the human eye.

The first indications that a wave theory alone could not account for the behavior of light appeared in three experiments: blackbody radiation, the photoelectric effect, and the Compton effect. Together, these experiments supported the hypothesis that light was made of particles, now called photons, each with energy $E = hf$ and momentum $p = hf/c$, where f is the frequency of the light involved and h = Planck's constant = 6.63×10^{-34} J·s. At this point, although the properties of photons simply were inferred from these experiments, the success of the photon model motivated physicists to search for a more fundamental theory from which photons and their properties could be derived.

Outline of the Theory

The foundations of the quantum (or photon) theory of light, that is, of QED, were established in the period from 1926 to 1940 by Dirac, Heisenberg, Pauli, Enrico Fermi, and Jordan. What emerged was a theory whose starting point is the classical treatment of electromagnetic fields based on Maxwell's equations and Hamilton's approach to the derivation of such equations of motion from a function expressing the energy density at each point in space, the Hamiltonian density. The quantity $H_{free} = (\mathbf{E}^2 + \mathbf{B}^2)/8\pi$ is the total energy per unit volume (at position **r** and time t) of the electromagnetic field in the absence of charges and currents. H_{free} is the Hamiltonian density for the free electromagnetic field, and **E** and **B** are the electric and magnetic field vectors at the point $(\mathbf{r},t)$. As in Maxwell's treatment, the **E** and **B** fields are then expressed in terms of the electromagnetic potentials φ and **A** since, in the Hamiltonian formalism, the interaction between the electromagnetic field and the current and charge densities **j** and ρ is expressed by the interaction energy $H_I = -(\rho\varphi - \mathbf{j} \cdot \mathbf{A}/c)$. It is found that if H_{free} is written in terms of the Fourier transforms of φ and **A**, then the Fourier coefficient for each frequency obeys the differential equation for a simple harmonic oscillator. Consequently, the Hamiltonian density for the free electromagnetic field is found to be equivalent to that of a system of uncoupled harmonic oscillators. It is at this point that we quantize the fields. That is, we assume that each harmonic oscillation is a quantum oscillator with a discrete set of energies $E = nhf$, which we interpret as a given number n of photons, each with energy hf and momentum hf/c. (In more technical terms, we treat the Fourier coefficients as quantum mechanical operators whose action is to create and destroy photon.) Thus, the original goal of deriving photons from a more fundamental basis is achieved.

Once the free electromagnetic field has been quantized, the next step is to treat its interaction with charged particles. The total Hamiltonian density is written as $H = H_{free} + H_{el} + H_I$, where H_{el} is the Hamiltonian density for the free, quantized electron (or any lepton) field; that is, the field obtained from the Dirac equation in the absence of the electromagnetic interaction. Unfortunately, exact solutions corresponding to H have not been found, so an approximation method is used to obtain numerical predictions that can be compared with experiment. A perturbation series, called the *S*-matrix expansion, is derived in which each successive term involves higher powers of the electron's charge $e = 1.6 \times 10^{-19}$ C, or more precisely, higher powers of the dimensionless constant $\alpha = 2\pi e^2/hc$ (called the fine-structure constant), which is approximately equal to 1/137. Because the Coulomb force between two charges is proportional to e^2, both e and α are referred to as coupling constants, since each is a measure of the strength with which the electromagnetic interaction couples one charge to another. Terms in the perturbation series are referred to as being first-order, second-order, and so on, according to the power of α they involve. By evaluating terms in the *S*-matrix expansion, predictions can be made that are accurate to a given power of α and then compared with experiment. As the experimental values become more precise, the theoretical predictions are made more precise by calculating terms in the series involving higher powers of α. For example, as we discuss below, the best theoretical value of the magnetic moment of the electron is obtained by evaluating all the terms in the perturbation series up to and including α^4. (Evaluating just the terms of order α^4 requires 891 Feynman diagrams, each involving about one hundred thousand terms.)

New Insights into Nature

Every new major theory illuminates aspects of nature that were not recognized previously. For example, special relativity introduced (among other things) the conversion of mass into energy and energy into mass via $E = \gamma mc^2$, and the Dirac equation for the relativistic electron introduced antiparticles. Perhaps the main new feature of QED is the photon and the way it participates in the electromagnetic interaction as the mediator of the electromagnetic force between charges. Before the mid 1800s, the electromagnetic interaction was viewed as a Newtonian force acting over the distance between charges. Next, in Maxwell's theory, electric and magnetic fields were viewed as existing at each point in space and the force on a charged particle was attributed to the electromagnetic field at the point the charge occupied. In QED, the electromagnetic interaction is viewed as resulting from an exchange of photons between charges; that is, photon exchange replaces the Maxwellian electromagnetic field as the source of the electromagnetic interaction. This picture, of an interaction being the result of an exchange of mediating particles, has been successfully extended to describe the weak and strong interactions and forms the current view of how these interactions take place.

QED also resolves the wave–particle duality of light, that is, the fact that in some experiments light behaves like a wave while in others it behaves like a particle. Since all experiments now are described in terms of the exchange of photons, it appears that this resolution is accomplished through a more comprehensive particle theory. Yet the photon of QED is not a particle in the usual (classical) sense. For example, it does not have a precise spacetime trajectory; it is not a "thing" with spatial extent and nonzero (rest) mass; it does not move with any speed other than the speed of light; and two photons with the same energy, momentum, and polarization are indistinguishable from each other. Basically, a photon is simply one unit of the electromagnetic field possessing a definite energy, momentum, and polarization.

Finally, QED alters our view of the vacuum, which classically was viewed as simply the empty state. Although the expectation values of **E** and **B** in the vacuum state are both zero, the expectation values of $\mathbf{E}^2$ and $\mathbf{B}^2$ are nonzero. This means that fluctuations of the vacuum can occur. Indeed, as we discuss in the next section, fluctuations of the vacuum are verified experimentally in the sense that they contribute a significant amount to effects observed in the lab. Furthermore, in the presence of an external (classical) electromagnetic field, these fluctuations create particle-antiparticle pairs that polarize like the constituents of a dielectric medium. This effect is called polarization of the vacuum, and it has been confirmed experimentally. Thus, our concepts of light and the vacuum both are radically altered by QED.

Experimental Tests of QED

Magnetic moments. Electrons, positrons, protons, muons, and so on all have intrinsic properties called mass, charge, and spin. Although spin had to be introduced in an ad hoc fashion to nonrelativistic quantum theory, Dirac showed that it is an automatic consequence of a quantum theory consistent with special relativity. In addition, Dirac's theory predicted that the spin **S** of an electron is related to its magnetic moment $\boldsymbol{\mu}$ by the relation $\boldsymbol{\mu} = (e/2mc)[\mathbf{L} + g\mathbf{S}]$, where m is the mass of the electron, **L** is its orbital angular momentum, and the constant g (called the electron's gyromagnetic ratio) is exactly two. However, this value of $\boldsymbol{\mu}$ results from treating the electron as a quantum field and the electromagnetic field as a classical field. In 1947 Kusch and Foley found experimental evidence that g actually is slightly larger than two. This result stimulated Hans A. Bethe, Schwinger, Feynman and others to account theoretically for the discrepancy, known as the anomalous magnetic moment of the electron, by using the quantized electromagnetic field and the Hamiltonian density H discussed above. They found that QED predicted a g-factor slightly larger than two, with the precise numerical value dependent upon how many terms in the perturbation series are evaluated. Each term in the series is interpreted as the degree to which the electron interacts with its own electromagnetic field, and the higher the order of the term, the less of a contribution it makes to the numerical value of the electron's magnetic moment (at least in the terms evaluated so far).

The most recent experimental values of the g-factor for the electron, $g(e^-)$, and its antiparticle the positron, $g(e^+)$, were measured by suspending each in a Penning trap:

$$g(e^-) = 2[1.0011596521884(43)]$$

$$g(e^+) = 2[1.0011596521879(43)],$$

where the value in parentheses means that the last two decimal places have an experimental uncertainly of ± 43 (43 parts in 10^{13}).

The best theoretical value of $g(e^-)$ at the present time, obtained through a collaboration of several groups, is accurate through terms of order α^4. Using the best current value of the fine-structure constant, $\alpha^{-1} = 137.0359979\ (32)$, they found

$$g(e^-) = 2[1.00115965214(4)],$$

with the same value predicted for the positron. Most of the uncertainty in the theoretical value results from the current experimental uncertainty in the fine-structure constant. However, the agreement between theory and experiment, to within 0.0001 ppm, is still spectacular. As Feynman notes (1985, p. 118): "This accuracy is equivalent to measuring the distance from Los Angeles to New York, a distance of over 3,000 miles, to within the width of a human hair."

There also is excellent agreement (to within 0.01 ppm) between the measured magnetic moments of positive and negative muons and the theoretical values resulting from evaluating terms in the perturbations series up to and including order α^5.

Lamb shift. Both the Dirac and Schrödinger theories predict that the $2S_{1/2}$ and the $2P_{1/2}$ states in hydrogen will have the same energy. However, in 1947, Lamb and Retherford found experimentally that the energy of the $2S_{1/2}$ state is higher than that of the $2P_{1/2}$ level by about 4.4×10^{-6}eV, which results in light being emitted with a frequency of about 1,060 MHz. To put this number in perspective, recall that the Schrödinger equation predicts the energy levels of an electron in hydrogen to be $(-13.6\ \text{eV})/n^2 = -\alpha^2 mc^2/2n^2$, where n is any nonnegative integer. So atomic transitions that end in the $n = 2$ state correspond to light being emitted with frequencies around 10^8 MHz. There are several corrections to these simple calculations of atomic energy levels when one includes effects of high order in α. The fine-structure correction results from including the first-order relativistic correction to the electron's kinetic energy and the interaction between the electron's spin and the magnetic field it experiences as a result of its motion relative to the proton. The energy levels are shifted by about 4.5×10^{-4} eV when $n = 2$, a shift of order $\alpha^4 mc^2/2n^2$, which causes the frequency of light emitted to be shifted by about 10^4 MHz. The hyperfine correction, which results from including the interaction between the spins of the electron and proton, is about 10^{-7} eV, or of order $\alpha^4(mc^2/2n^2)(m/m_p)$, where m_p is the proton's mass, 1,836 m. Thus, the Lamb shift in the $n = 2$ level is intermediate between the fine and hyperfine corrections and is about 10 percent of the fine-structure correction. (It was from this particular analysis that α received its name, since it provided a convenient measure of the relative scale of the corrections that accounted for the fine structure that already had been observed in spectral lines.)

As with the magnetic moments discussed above, the Lamb shift is accounted for when the quantized electromagnetic field is used in place of the classical one. The best experimental values for the Lamb shift in hydrogen, given in terms of the frequency of light emitted in the transition, are

$$f(2S_{1/2} - 2P_{1/2}) = 1{,}057.845(9)\text{ MHz}$$

and

$$1{,}057.851(2)\text{ MHz.}$$

The best theoretical values for the Lamb shift are

$$f(2S_{1/2} - 2P_{1/2}) = 1{,}057.853(13)\text{ MHz}$$

and

$$1{,}057.871(13)\text{ MHz,}$$

with the first value resulting when the root-mean-square (rms) radius of the proton is taken to be 0.805 (11) fm, and the second when it is taken to be 0.862 (12) fm. Experiment and theory agree to within 10 ppm if the first (older) value of the proton's radius is used, but they are in less satisfactory agreement (of about 20 ppm) if the second (newer) value is used. The main difficulty is in determining the rms radius of the proton rather in than the theory of QED itself.

The terms of the perturbation series that contribute to the Lamb shift can be described as follows: first, the interaction of the electron with its own electromagnetic field (the self-energy correction), which leads to the modification in the electron's magnetic moment described above; second, the spontaneous production of electron-positron pairs in the neighborhood of the nucleus, which polarize and thus partially screen the electron from the proton's charge (the vacuum polarization correction); third, fluctuations of the vacuum, which alter the motion and kinetic energy of the electron; and fourth, corrections involving the proton's size, charge distribution, mass, and motion.

Lamb shifts also have been measured in other transitions in hydrogen, as well as in deuterium, hydrogen-like atoms, helium, helium-like atoms, positronium, and muonium. All of these results are in excellent agreement with the theoretical predictions.

Hyperfine structure. As mentioned above, an additional correction to the spectrum of hydrogen results when the interaction between the spins of the electron and proton is taken into account since the electron has one energy if its spin is parallel to the spin of the proton (spin-1 hydrogen) and another if their spins are anti-parallel (spin-0 hydrogen). The frequency of light emitted between these two levels in the ground state of hydrogen is the most accurately measured number in physics:

$$f = 1{,}420.4057517667(9)\text{ MHz.}$$

The current, most accurate, theoretical value of the this splitting is

$$f = 1{,}420.45199(14)\text{ MHz,}$$

with the main uncertainties coming from the experimental uncertainty in the fine-structure constant and the lack of precise knowledge about the structure of the proton. Thus, theory and experiment currently agree to about 30 ppm.

The hyperfine splitting between the spin-0 and spin-1 levels in the ground state of muonium also has been measured very accurately and found to be

$$f = 4{,}463.30288(16)\text{ MHz.}$$

Since muonium is essentially an electrodynamic system, with the weak and strong interactions making almost no contribution, the comparison between theory and experiment provides an especially rigorous test of QED. Currently, the best theoretical value is

$$f = 4{,}463.303(2)\text{ MHz,}$$

and the agreement with experiment, to within 1 ppm, is excellent. The uncertainty in the theoretical value arises from the uncertainty in the muon's mass, and the uncertainty in the value of the fine-structure constant.

Measurements of hyperfine splitting also have been made in higher energy levels of hydrogen and in positronium, deuterium, and tritium. All are found to be in excellent agreement with the theoretical predictions when the proper nuclear structure and recoil corrections are taken into account.

Other tests. The predictions of QED also have been tested in many other experiments, such as in high energy electron-electron and electron-positron scattering, where they agree to within 1 percent with theoretical calculations made through order α^3, in the scattering of light by light (Delbruck scattering), in electron-positron pair production and pair-annihilation, in the Casimir effect (in which two neutral metal plates experience a slight attraction due to fluctuations of the vacuum), and so on. At present, there is no known discrepancy between experiment and theory, and the most precise theoretical predictions and experimental measurements are in excellent agreement.

Difficulties with the Theory

The main difficulty of the QED theory is that when higher terms in the perturbation series are evaluated, some of the resulting integrals are divergent (that is, infinite). However, these infinities can be isolated and removed (in every order of perturbation theory) by redefining the charge and mass parameters that appear in the theory. In principle, even if the infinities did not occur, it still would be necessary to renormalize the charge and mass parameters because of the way the Hamiltonian density H is divided up in the perturbation approach. More specifically, the Hamiltonian densities for the free electromagnetic field H_{free} and the free electron field H_{el} together form the unperturbed system, and the interaction term H_I is considered as the perturbation. The mass parameter m appears in the unperturbed Hamiltonian, so by virtue of using a perturbation approach, it represents the mass an electron would have in the absence of the electromagnetic interaction. For this reason, it is called the bare mass. The physical (or dressed) mass is then the bare mass plus that part of the mass resulting from the interaction of the electron with its own electromagnetic field. Neither the bare mass nor its interaction correction are observable; the only observable is their sum, the physical mass. Consequently, each can be calculated separately, and even though each involves integrals that diverge, all that matters is that when they are added together to give the physical mass, the divergent integrals cancel. Thus, the diverging terms disappear in the process of renormalizing the mass.

Exactly the same considerations apply to the electron charge. The parameter e in the theory is the bare charge; that is, the charge the electron would have in the absence of the electromagnetic interaction. This charge is then renormalized so that the physical (or dressed) charge is identified as the sum of the bare charge and the charge arising from the interaction of the electron with its own electromagnetic field. As in the renormalization of mass, the diverging integrals in the bare charge and its correction cancel when the two are added together to obtain the physical charge.

Most physicists now accept the renormalization procedure, since it leads to predictions that are in excellent agreement with experiment and since modern methods have made it more rigorous. However, many continue to think that subtracting infinities is not a valid mathematical procedure, no matter how elegantly it is performed. To these physicists, the renormalization process represents a serious problem in the fundamental structure of the theory. Other physicists think that the success of renormalization in producing numbers verified by experiment implies that it is, or someday will be, justified. Still others think that the divergences might be removed when QED is incorporated into a more comprehensive quantum field theory that includes gravity as well as the strong and weak interactions.

A second difficulty with QED is that no one has been able to show that the perturbation series converges, or that if it converges, it converges to the correct limit. In fact, in 1952 Dyson argued that the series probably diverges and that at best it is an asymptotic series, that is, a series whose first few terms approach a limit but which then diverges as more terms are added. More specifically, he argued that if the coupling constant is a fixed number, then the series may not converge, but if the coupling constant is considered as a variable, then as it tends to zero, increasingly long initial segments of the expansion should converge to the right limit. This would explain why the sum of the first few terms is in such good agreement with experiment. However, Dyson's argument is not a rigorous mathematical proof, so it is not regarded as conclusive (even though he found it so convincing that he left the field to pursue research in other areas).

Conclusion

Because of its spectacular agreement with experiment, QED is considered by many to be one of the most successful theories in physics. Although some difficulties remain, most physicists accept the theory as being basically correct. Furthermore, many features of QED have been incorporated with great success into the recent theories of strong, weak, and electroweak interactions, thus reinforcing the procedures and points of view upon which it is based and solving some of the difficulties of definition that occur for QED in isolation, but not for the combined theory.

See also: DIRAC, PAUL ADRIEN MAURICE; ELECTROMAGNETISM; FERMI, ENRICO; FEYNMAN, RICHARD; FEYNMAN DIAGRAM; HEISENBERG, WERNER KARL; HYPERFINE STRUCTURE; INTERACTION, STRONG; INTERACTION, WEAK; LAMB SHIFT; MAGNETIC MOMENT; MAXWELL'S EQUATIONS; PAULI, WOLFGANG; RADIATION, BLACKBODY; RELATIVITY, SPECIAL THEORY OF; RENORMALIZATION

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MARK D. SEMON

QUANTUM FIELD THEORY

Quantum field theory is the correct way to describe systems like the electromagnetic field in accordance with the laws of quantum mechanics (and any other restrictions, such as those due to relativity). It forms the basis of modern elementary particle physics and proves invaluable in other many-body problems like superconductivity and the physics of solids.

Let us begin with a single mass coupled to an anchored spring and vibrating along the x direction. This oscillator is a system with one degree of freedom, since only one coordinate, x is needed to describe it. A system of N masses would have N degrees of freedom. A field is a system with infinite number of degrees of freedom. For example, if we had an infinite array of masses in a line, coupled to their neighbors by springs, the displacements of each mass from equilibrium would constitute a field. Another example is the electromagnetic field whose degrees of freedom are the values of the electric and magnetic field at each point in space. Notice that the latter has an infinite number of degrees of freedom within any finite volume, and this causes many problems. The aim of quantum field theory is to describe how these degrees of freedom behave, following the laws of quantum mechanics.

We will work our way to the field by starting with the simplest system: a single mass m attached to a spring of force constant k. In classical mechanics this mass will vibrate with frequency $\omega = 2\pi f = \sqrt{k/m}$. It can have any finite amplitude x_0, and the corresponding energy will be $E = \frac{1}{2}kx_0^2$.

In the quantum version the oscillator can only have energy $E = \hbar\omega(n + \frac{1}{2})$, where $n = 0, 1, 2 \ldots$ and $\hbar = 1.05 \times 10^{-34}$ J·s is Planck's constant. Note that the lowest energy is not zero but $\frac{1}{2}\hbar\omega$, which is called the zero-point energy. The uncertainty principle in quantum mechanics, which forbids a state of definite location and momentum, does not allow the classical zero energy state in which the mass sits at rest in the equilibrium position. Next, the fact that the levels are equally spaced means that instead of saying the oscillator is in a state labeled by n, we can say that there are n quanta of energy $\hbar\omega$. This seemingly semantic point proves seminal as we shall see.

Now consider two masses m attached to springs of force constant k as in the Fig. 1. The coordinates x_1 and x_2 are coupled to each other and the motion is quite complicated. But consider the fol-