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impossible because such a decay would violate momentum conservation; however, the single photon decay of the $^3S_1$ state is possible if another particle is present to carry the recoil momentum. The probabilities of direct decays into four or five photons are much smaller by a factor of order α^2 (the fine structure constant squared) and have not yet been observed experimentally. The mean lifetimes of parapositronium and orthopositronium are about 0.1 ns and 140 ns, respectively.

Energy levels above the ground state are produced with much smaller probability, making studies of these levels more difficult. Photons emitted from the $n = 2$ to $n = 1$ transition, corresponding to the Lyman-α spectral line of hydrogen, were reported by Karl F. Canter, Allen P. Mills Jr., and Stephan Berko in 1975, along with a measurement of the fine-structure energy separation between the 2^3S_1 and 2^3P_2 states. In their experiment, positronium in the $n = 2$ state was produced by directing positrons from a ^{58}Co source onto a copper target inside a metal cavity. A radio-frequency field was applied to drive the transition from the 2^3S_1 state to the 2^3P_2 state, which occurred at 8,625 MHz, as predicted. The 2^3P_2 state decayed to the 1^3S_1 state with the emission of the 2,430Å Lyman-α photon, which was detected with a photomultiplier tube, and, finally, the 1^3S_1 state decayed to three gamma rays, which were detected with a scintillation counter. The results of this experiment were fully consistent with the predictions of QED.

Precision experimental work in the late twentieth century sparked renewed interest in positronium. Several experiments have measured decay rates of orthopositronium that are slightly higher, on the order of one part in a thousand, than predicted by theoretical estimates. Experiments of this kind are done with great care to identify and correct for any small effects that could influence the outcome of the measurements. This discrepancy has prompted refined calculations of higher-order corrections and even led to speculations about possible exotic particles, as yet undetected, into which positronium might decay. As experiments are performed with greater accuracy and theoretical calculations are carried out with greater sophistication, this discrepancy may disappear. If the discrepancy persists, it would pose a serious problem for QED.

See also: POSITRON; QUANTUM ELECTRODYNAMICS; SPECTRAL SERIES

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ROGER A. ERICKSON

POTENTIAL

See CHEMICAL POTENTIAL; ELECTRIC POTENTIAL; ENERGY, POTENTIAL; IONIZATION POTENTIAL

POTENTIAL BARRIER

The term "potential barrier" is used when systems are described in terms of energy flow. For example, consider a simple pendulum as shown in Fig. 1. The pendulum has its greatest kinetic energy, that is, energy by virtue of motion, when it passes through its equilibrium position at $\theta = 0$ (straight down). The pendulum has its greatest potential energy, that is, energy by virtue of location, when it reaches its maximum height at $\theta = \pm\theta_0$. The sum of the kinetic and potential energies is the total energy, which is constant in this case. As the pendulum rises, its potential energy increases, and since the total energy remains constant, its kinetic energy must decrease. Because kinetic energy never can be negative, the pendulum must stop when its total energy becomes all potential. One way of expressing the fact that the potential energy cannot increase any further is to say that the pendulum has "run into a potential barrier." The point at which this occurs is called a turning point since the pendulum reverses its direction of motion there.

Potential barrier acquires a more visual meaning when the energy flow is illustrated graphically, in an energy diagram. The energy diagram representing the motion of a simple pendulum is shown in Fig. 2. The horizontal axis represents the angular displacement from equilibrium, with $\theta = 0$ denoting the equilibrium position (straight down) and $\theta = \pm\pi$ denoting the totally inverted position (straight up). The vertical axis represents energy. The solid curve shows the potential energy at each θ, the line of pluses (+++) shows the total energy at each θ, and the difference between them is the kinetic energy at each θ. The pendulum is represented by a point on the line of constant energy. At $\theta = 0$, the point representing the pendulum is on the line of constant energy above $\theta = 0$, and the energy is all kinetic. As the pendulum moves away from equilibrium in the direction of increasing θ, the point representing it remains on the line of constant energy and also moves in the direction of increasing θ. Since the total energy remains constant while the potential energy increases, the kinetic energy must decrease. Thus, the diagram shows that the pendulum must slow down as it moves away from equilibrium. The farther from equilibrium the pendulum moves, the smaller its kinetic energy becomes until the total energy is all potential at $\theta = \theta_0$, whence the pendulum stops. In the energy diagram, the point representing the system also stops at θ_0 and has the appearance of "running into a potential barrier," that is, into the potential energy curve. After hitting the potential energy curve at θ_0, the system point moves back along the line of constant energy in the direction from which it came until it hits the potential energy curve at $-\theta_0$, where it again stops and reverses its direction of motion. Thus, potential barrier acquires a more picturesque meaning when we watch the system point in an energy diagram bounce off the potential energy curve at each turning point, since its motion resembles that of a real elastic ball bouncing off real barriers at $\pm\theta_0$.

The simple pendulum is an example of a bound system, that is, of a system whose motion is confined to a bounded region. Potential barriers also occur for systems that are unbounded. For example, consider an elastic ball hitting a wall. Describing what happens in terms of forces, we say that the wall exerts a force on the ball, which eventually brings the ball to rest and reverses its direction of motion. Viewing this same process in terms of energy flow, we say that as the ball hits the wall, its kinetic energy decreases while its potential energy increases, since kinetic energy is being stored in the ball's deformation. When the ball comes to rest against the wall, we say it has "run into a potential barrier" because its total energy is all potential. Since the ball can proceed no further in its original direction of motion, it begins moving back in the direction from which it came. Once the ball moves away from the wall, its motion is unbounded since there is nothing in this direction to impede its travel. Thus, potential barriers also occur in unbound systems such as balls bouncing off walls, alpha particles colliding with heavy nuclei, carts rolling up hills, and so on.

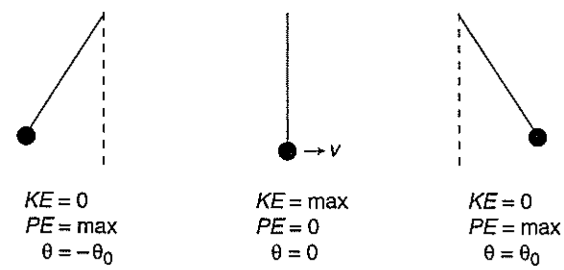

Figure 1 A simple pendulum. When the pendulum moves through its equilibrium position, the total energy is all kinetic. At the highest point in its motion, the total energy is all potential, and the pendulum stops since it has run into a potential barrier. (Courtesy of T. Jayaweera)

In general, one can use either the force or energy framework for describing and predicting motion.

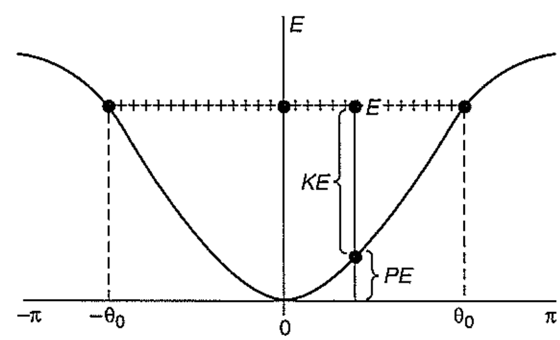

Figure 2 The energy diagram for the simple pendulum of Fig. 1. The point representing the pendulum moves along the line (+++) of constant energy E, and the pendulum's potential energy is denoted by the solid curve. The turning points of the motion, where the pendulum runs into potential barriers, are denoted by $\pm\theta_0$. (Courtesy of T. Jayaweera)

The energy description, however, has a distinct advantage: A great deal of information can be obtained from studying energy flow without actually having to solve any equations of motion. This simplifies many problems and, furthermore, means that studying energy flow provides a lot of information even when the equations of motion are too complicated to solve. Thus, the framework of energy is used throughout physics, and potential barriers and turning points play important roles in understanding many different types of physical systems.

Potential Barriers in Classical Mechanics

Studying the energy flow of a simple pendulum actually enables us to understand a number of different systems since the simple pendulum is just one example of a simple harmonic oscillator. Similarly, studying the potential energy function $V(r) = -\alpha/r$ enables us to understand many different gravitational and electrostatic systems. For example, when $\alpha = GMm$ (with $G = 6.67 \times 10^{-11}$ in SI units), $V(r)$ is the gravitational (Kepler) potential energy of a mass m a distance r from a second mass M fixed at $r = 0$. When $\alpha = -kQq$ (with $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9$ in SI units and 1 in cgs units), $V(r)$ is the electrostatic (Coulomb) potential energy of a charge q a distance r from a second charge Q fixed at $r = 0$. For motion in three dimensions in such a potential, the total potential energy function depends only on r and is given by

$$U(r) = \frac{L^2}{2mr^2} - \frac{\alpha}{r},$$

where L is the total angular momentum of the particle, which, for our (central) potential $V(r)$, is constant. The extra term $(L^2/2mr^2)$ originates in the rotational kinetic energy, but because r is the only variable it contains, it behaves like a potential energy keeping particles with nonzero angular momentum away from the center of force at $r = 0$; thus, we study the radial motion with this combined potential energy. A graph of the various potential energies is shown in Fig. 3, with the dash-dot-dash curve representing $(-\alpha/r)$, the dashed curve representing $(L^2/2mr^2)$, and the solid curve representing their sum $U(r)$.

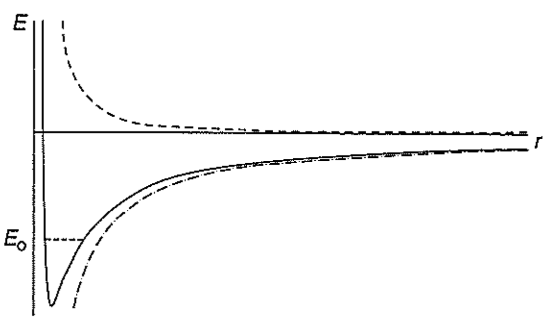

Figure 3 The energy diagram for a bound mass or charge ($E < 0$). (Courtesy of T. Jayaweera)

As mentioned above, the real power of the energy diagram lies in the amount of information that can be obtained from it without having to solve any equations of motion. For example, looking more closely at Fig. 3, we note that $U(r)$ has a minimum, and by differentiating $U(r)$ and setting the result equal to zero, we find that this minimum occurs at $r = L^2/m\alpha$. Thus, whenever the total energy equals $-m\alpha^2/2L^2$, the particle moves at a fixed distance $r = L^2/m\alpha$ from the center of force; that is, the particle moves in a circular orbit with a constant speed. If the total energy is greater than this value but still less than zero, the particle's radial motion will be bounded between the two turning points $r_0 = r_{\max}$ and $r_0 = r_{\min}$. At these points the total energy will be all potential and will be given by

$$E_0 = \frac{L^2}{2mr_0^2} - \frac{\alpha}{r_0}.$$

Multiplying both sides of this equation by r_0^2 and using the quadratic formula, we find that

$$r_{\max} = -\beta\left[1 + \left(1 + \frac{L^2}{2m\beta^2 E_0}\right)^{1/2}\right]$$

and

$$r_{\min} = -\beta\left[1 - \left(1 + \frac{L^2}{2m\beta^2 E_0}\right)^{1/2}\right],$$

with $\beta = \alpha/2E_0$ (remember, $E_0 < 0$). Since bounded, periodic motion between two fixed radii corresponds to motion in an ellipse, the energy diagram shows us that whenever the total energy is between $-m\alpha^2/2L^2$ and zero, the particle will move in an elliptical orbit. Furthermore, from our knowledge of conic sections, we conclude that the semi-major axis of this ellipse is $a = (r_{max} + r_{min})/2$ and its eccentricity is $e = (r_{max} - r_{min})/(r_{max} + r_{min})$.

When the total energy is greater than zero and $V(r)$ is a repulsive potential ($\alpha < 0$), the particle will run into only one potential barrier, and this turning point will be the closest an unbound particle with angular momentum L can get to the center of force at $r = 0$. This particular potential barrier is called the centrifugal barrier since, for small r, the main contribution to it comes from the term $L^2/2mr^2$ whose gradient is the centrifugal force (a fictional force that occurs when we apply Newtonian mechanics in rotating reference frames).

Thus, a great deal of information about the three-dimensional motion of a particle with potential energy $V(r)$ can be deduced from an energy diagram using relatively simple mathematics. The same type of analysis can be carried out with other important physical problems by defining the relevant potential energy functions, such as the Yukawa potential energy, the general relativistic and perturbation theory corrections to the Kepler potential energy α/r, the potential energy of a spinning, symmetric top, the Morse potential energy function for an atom in a diatomic molecule, the isotropic oscillator potential energy $V(r) = kr^2/2$ for the vibrational motion of a diatomic molecule, the van der Waals potential energy, and so on.

Potential Barriers in Quantum Mechanics

A number of interesting potential barriers occur in (nonrelativistic) quantum mechanics, where energy diagrams are used to understand some of the more surprising features of quantum systems. The most famous example is that of alpha decay, in which a nucleus emits an alpha particle and changes into another nucleus. In 1928 George Gamow and, independently, Edward U. Condon and Ronald W. Gurney showed that alpha decay could be understood as an alpha particle inside a nucleus tunneling through the potential barrier it encounters at the nuclear edge. Figure 4 shows an approximation of the potential energy of such an alpha particle with total energy E_α. Classically, the alpha particle could not move beyond the nuclear radius at $r = a$ since it would run into a potential barrier there. But in quantum mechanics, the alpha particle is described by a wave function ψ, and its behavior at the nuclear edge can display properties not normally associated with particles. In particular, ψ is nonzero at the classical turning point $r = a$ and decays exponentially just beyond it in the classically forbidden region $a < r < b$. (This region is classically forbidden because to be in it the particle would have to have a potential energy greater than its total energy, or a negative kinetic energy.) Consequently, there is a small but finite probability of finding the alpha particle outside the nucleus. In other words, if the height of the barrier is not too much greater than the alpha particle's energy E_α, and if the barrier's width $(b - a)$ is not too large, then ψ can be nonzero in the classically forbidden region where $V(r) > E$. A nonzero ψ in this region results in ψ being nonzero for $r > b$. Although the probability of finding an alpha particle at points $r > b$ is quite small, nonetheless, since the alpha particle encounters the nuclear barrier quite frequently (about 10^{21} times per second), quantum theory predicts that some experiments will find it outside the nucleus. In fact, treating alpha decay as tunneling through a potential barrier (also called barrier penetration), correctly predicts all the results found experimentally.

Analogs to quantum tunneling occur in a variety of classical wave systems. For example, in classical optics, a light wave traveling through glass can be directed so that it hits a glass-air interface at an angle greater than the critical angle. In this case, all of the wave will be totally internally reflected. However, if a second piece of glass is put close to the first, then a small part of the wave will tunnel through the air

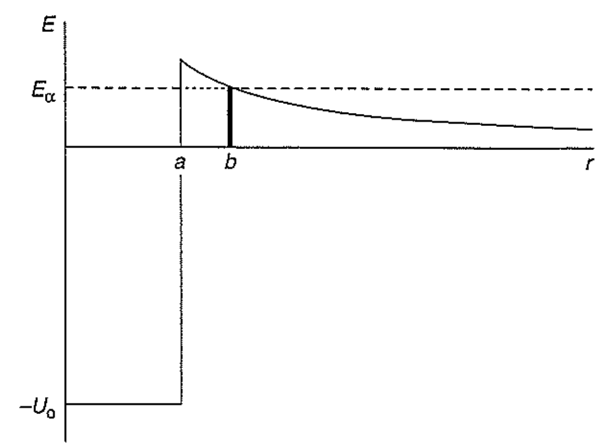

Figure 4 The energy diagram for an alpha particle inside a nucleus of radius a. E_α is the energy of the alpha particle, and $(b - a)$ is the width of the potential barrier. (Courtesy of T. Jayaweera)

barrier between them and appear in the second piece of glass. This phenomenon, called frustrated total internal reflection, was first observed by Isaac Newton around 1700 and now is routinely demonstrated in undergraduate physics labs. Similar phenomena also occur with other types of waves.

Tunneling through potential barriers also provides the basis for our understanding of spontaneous nuclear fission, of how electrons pass through thin oxides and insulators (e.g., tunnel diodes), of the dynamics of the ammonia molecule and the ammonia maser, of the Josephson effect and superconducting quantum interference devices, and of Scanning Tunneling Microscopes.

See also: BARRIER PENETRATION; DECAY, ALPHA; ENERGY, KINETIC; ENERGY, POTENTIAL; JOSEPHSON EFFECT; MASER; MOMENTUM; PENDULUM; SCANNING TUNNELING MICROSCOPE; SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE; VAN DER WAALS FORCE

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MARK D. SEMON

POTENTIOMETER

A potentiometer is an electrical device consisting of a length of resistance wire (or resistive material) with some resistance R and a tap point that slides on this length of resistance wire, making electrical contact with it. There are three electrical connections to a potentiometer: points A and B at the ends of the resistance wire and the sliding tap point T (see Fig. 1). The volume control of a radio or stereo is a simple, inexpensive potentiometer; in contrast, a precision potentiometer is an expensive instrument that can be used to measure voltage to high precision.

If connections are made only to the tap point T and to point A of a potentiometer, it becomes an adjustable resistor, or rheostat. The resistance between points T and A is indicated by R_1, and this varies from 0 to R as the slider is moved over the length of resistance wire from the end near point A to the end near point B.

The potentiometer gets its name from the fact that it can "meter out" varying amounts of electrical potential, or voltage, from the output between the tap point T and one end of the potentiometer wire (e.g., A). Suppose a battery with emf V is connected between end points A and B as shown in Fig. 1. Let the resistance between B and T be R_2 while the resistance between T and A is R_1. These two resistances constitute a voltage divider so that the voltage between tap T and end A (V_{TA}) is a fraction of the voltage between B and A (V_{BA}). We have that $V_{TA} = [R_1/(R_1 + R_2)]V_{BA} = (R_1/R)V_{BA}$. The resistance ($R_1 + R_2$) is a constant equal to the resistance R of the potentiometer. As the slider is moved along the resistance wire, resistance R_1 varies from 0 to R, and the voltage V_{TA} between the tap and A varies from 0 to V_{BA}. This is the basic way to produce a variable amount of voltage from a fixed voltage.

In the case of a radio volume control, the voltage V_{BA} presented to the potentiometer is an audio-frequency voltage corresponding to the sound wave. The variable amount of voltage from the potentiometer tap (V_{TA}) is sent (through an amplifier) to

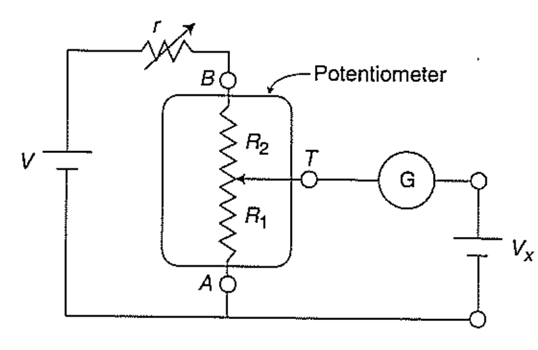

Figure 1 Schematic diagram of a potentiometer. The potentiometer is shown connected with working voltage V, rheostat r, and galvanometer G to measure voltage V_x.