Basic Functions and Their Inverses

Definition. A function is a rule that assigns to every x value in the domain, one and only one y value in the range.

Definition. A function is **one-to-one** if for every y value in the range, there is one and only one x value such that $f(x) = y$.

Definition. Inverse Function: Suppose $f(x)$ is a one-to-one function with domain D and range R. The inverse function $f^{-1}(x)$ is defined by

$$f^{-1}(b) = a \text{ if } f(a) = b$$

The domain of $f^{-1}(x)$ is R and the range of $f^{-1}(x)$ is D.

Finding an Inverse: $f^{-1}(x)$ is a reflection of $f(x)$ through the line $y = x$. To calculate $f^{-1}(x)$:

1. Solve the equation $y = f(x)$ for x. This gives a formula $x = f^{-1}(y)$ where x is expressed as a function of y.
2. Interchange x and y to obtain the expression $y = f^{-1}(x)$

Some Standard Functions

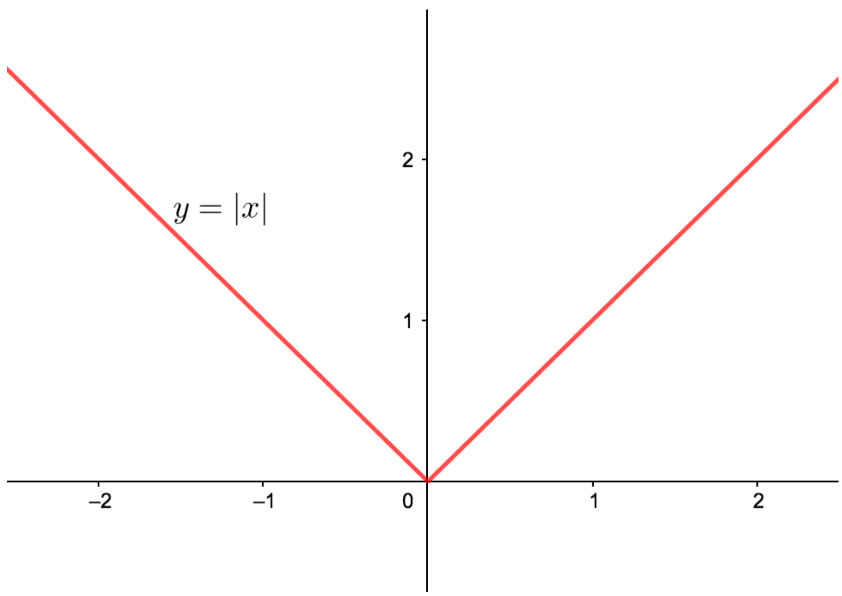

Figure 1: **The Absolute Value Function**

Note: This function is NOT one-to-one so it does not have an inverse. However, you could invert the section where $x \geq 0$ or the section $x \leq 0$

Some Standard Functions and Their Inverses

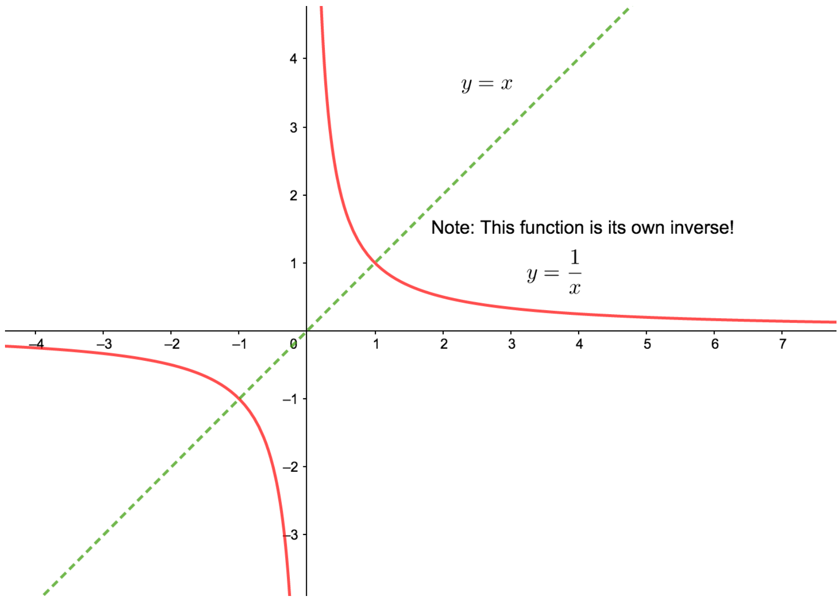

Figure 2: **The Reciprocal Function**

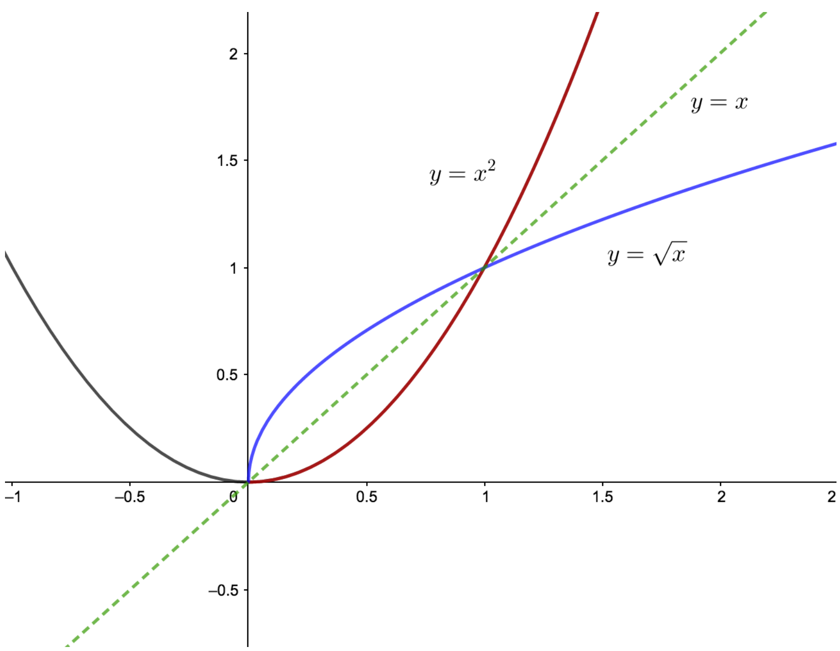

Figure 3: **The Quadratic Function and its Inverse**

Some Standard Functions and Their Inverses

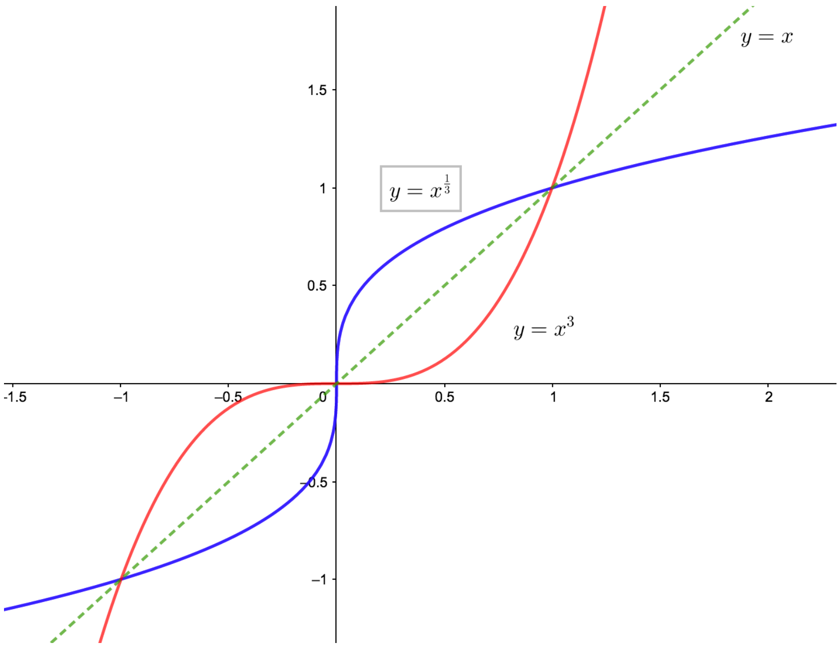

Figure 4: **The Cubic Function and its Inverse**

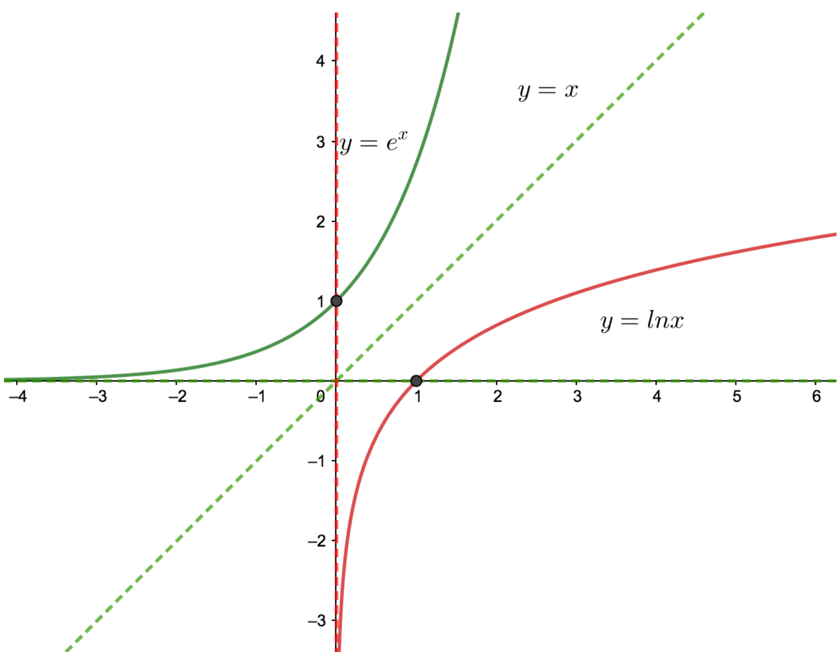

Figure 5: **The Exponential Function and the Natural Log**

Some Standard Functions and Their Inverses

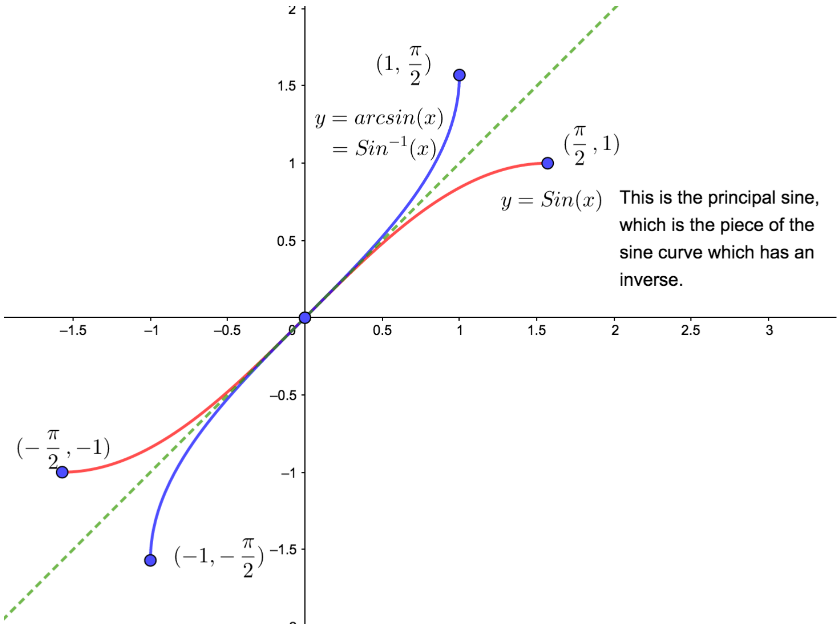

Figure 6: **The Sine Function and its Inverse**

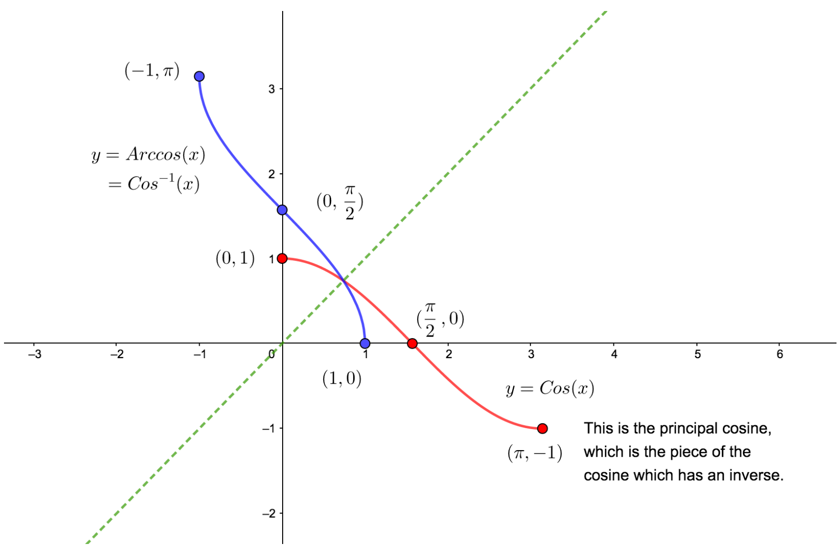

Figure 7: **The Cosine Function and its Inverse**

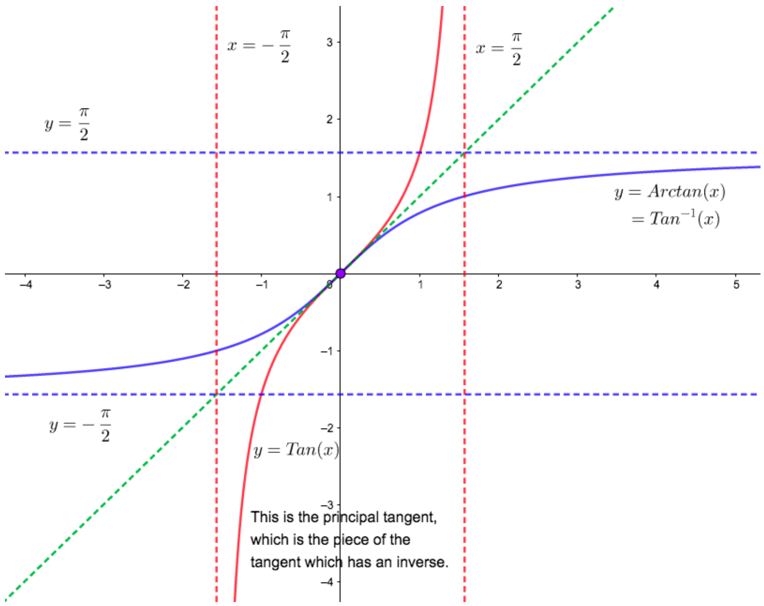

Figure 8: **The Tangent Function and its Inverse**

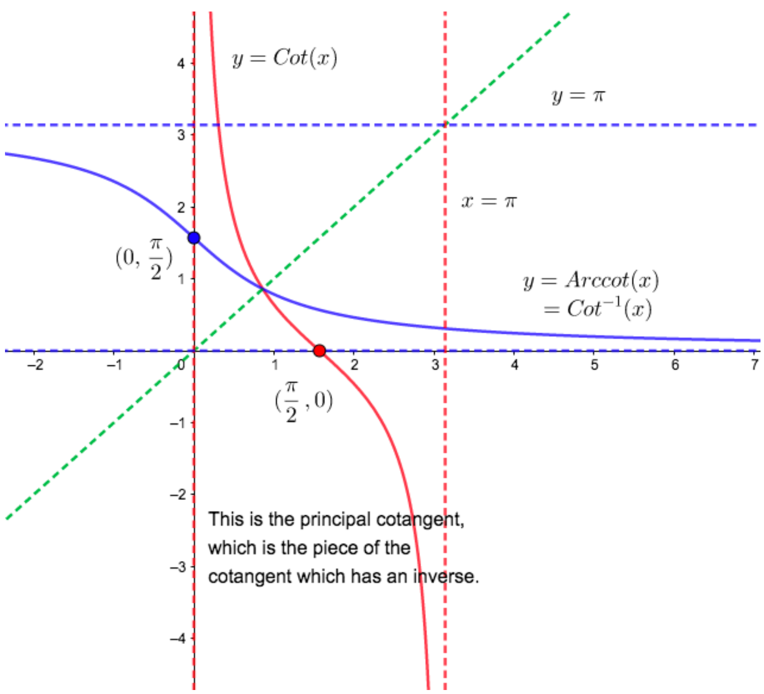

Figure 9: **The Cotangent Function and its Inverse**

Some Standard Functions and Their Inverses

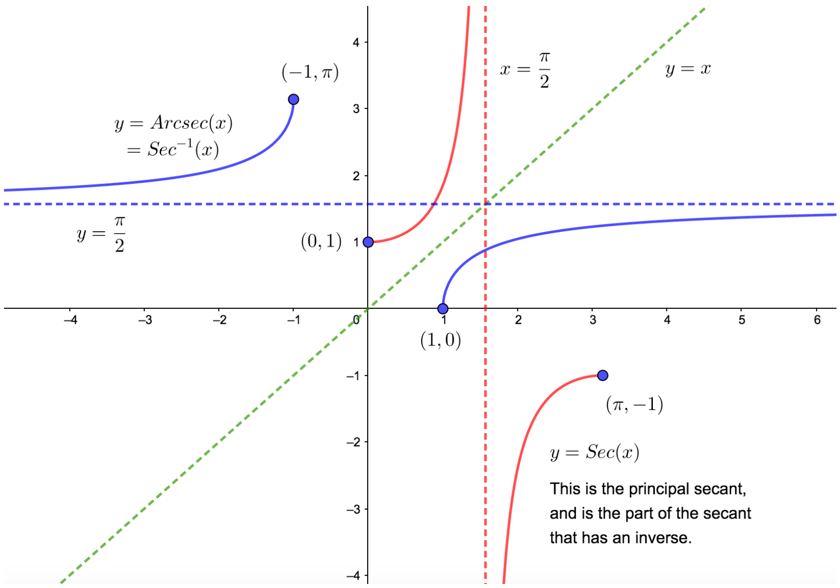

Figure 10: **The Secant Function and its Inverse**

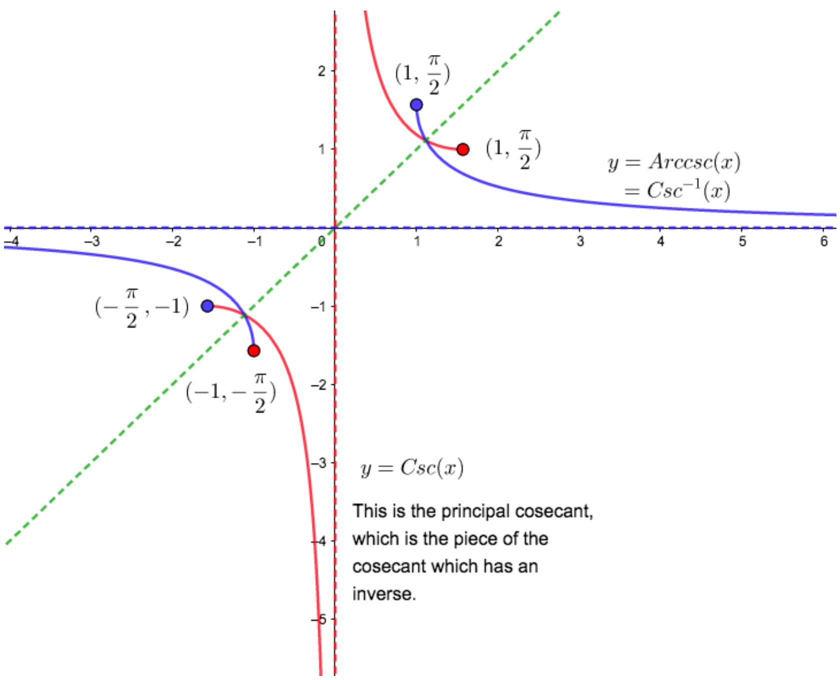

Figure 11: **The Cosecant Function and its Inverse**