Manipulating Rational Functions Practice

Example 1. $\dfrac{1}{x+2}+\dfrac{1}{x-1}$

Solution.

$$\begin{aligned}\frac{1}{x+2}+\frac{1}{x-1} &= \frac{(x-1)}{(x-1)}\cdot\frac{1}{x+2}+\frac{(x+2)}{(x+2)}\cdot\frac{1}{x-1}\\ &= \frac{(x-1)}{(x-1)(x+2)}+\frac{(x+2)}{(x-1)(x+2)}\\ &= \frac{x-1+x+2}{(x-1)(x+2)}\\ &= \frac{2x+1}{(x-1)(x+2)}\end{aligned}$$

Example 2. $\dfrac{x^{-1}+y^{-1}}{x^{-2}+y}$

Solution.

$$\begin{aligned}\frac{x^{-1}+y^{-1}}{x^{-2}+y} &= \frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2+y}}\\ &= \frac{\frac{y}{y}\cdot\frac{1}{x}+\frac{x}{x}\cdot\frac{1}{y}}{\frac{1}{x^2}+\frac{x^2}{x^2}\cdot y}\\ &= \frac{\frac{y+x}{xy}}{\frac{1}{x^2}+\frac{x^2y}{x^2}}\\ &= \frac{\frac{x+y}{xy}}{\frac{1+x^2y}{x^2}}\\ &= \frac{x+y}{xy}\cdot\frac{x^2}{1+x^2y}\\ &= \frac{x+y}{y}\cdot\frac{x}{1+x^2y}\\ &= \frac{x^2+xy}{y+x^2y^2}\end{aligned}$$

Example 3. $\dfrac{x+y}{x^{-1}+y^{-1}}$

Solution.

$$\begin{aligned}
\frac{x+y}{x^{-1}+y^{-1}} &= \frac{x+y}{\frac{1}{x}+\frac{1}{y}} \\
&= \frac{x+y}{\frac{y}{y}\cdot\frac{1}{x}+\frac{x}{x}\cdot\frac{1}{y}} \\
&= \frac{x+y}{\frac{y+x}{xy}} \\
&= \frac{x+y}{1}\cdot\frac{xy}{x+y} \\
&= xy
\end{aligned}$$

Example 4. $\dfrac{\sqrt{x-3}+2(x-3)^{-1}}{\sqrt{(x-3)^3}+1}$

Solution.

$$\begin{aligned}
\frac{\sqrt{x-3}+2(x-3)^{-1}}{\sqrt{(x-3)^3}+1} &= \frac{(x-3)^{\frac{1}{2}}+\frac{2}{(x-3)}}{(x-3)^{\frac{3}{2}}+1} \\
&= \frac{\frac{(x-3)}{(x-3)}\cdot\frac{(x-3)^{\frac{1}{2}}}{1}+\frac{2}{(x-3)}}{(x-3)^{\frac{3}{2}}+1} \\
&= \frac{\frac{(x-3)^{\frac{3}{2}}}{(x-3)}+\frac{2}{(x-3)}}{(x-3)^{\frac{3}{2}}+1} \\
&= \frac{\frac{(x-3)^{\frac{3}{2}}+2}{(x-3)}}{(x-3)^{\frac{3}{2}}+1} \\
&= \frac{(x-3)^{\frac{3}{2}}+2}{(x-3)^{\frac{5}{2}}+(x-3)}
\end{aligned}$$